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On the Geometry of a Go-Board

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The articles [15], [2], [8], [1], [16], [13], [5], [3], [9], [10], [14], [17], [4], [6], [7], [11], and [12] provide the notation and terminology for this paper.

For simplicity, we follow the rules: i, j, n denote natural numbers, r, s, r_1, s_1, r_2, s_2 denote real numbers, p denotes a point of $\mathcal{E}^2_{\rm T}$, and G denotes a Go-board.

Next we state a number of propositions:

(4)[1] Let M be a non empty Reflexive metric structure, u be a point of M, and r be a real number. If $r > 0$, then $u \in \mathrm{Ball}(u,r)$.

(6)[2] For every subset B of $\mathcal{E}^n_{\rm T}$ and for every point u of $\mathcal{E}^n$ such that $B = \mathrm{Ball}(u,r)$ holds B is open.

(7) Let M be a non empty metric space, u be a point of M, and P be a subset of $M_{\rm top}$. Then $u \in \mathrm{Int}\,P$ if and only if there exists a real number r such that $r > 0$ and $\mathrm{Ball}(u,r) \subseteq P$.

(8) Let u be a point of $\mathcal{E}^n$ and P be a subset of $\mathcal{E}^n_{\rm T}$. Then $u \in \mathrm{Int}\,P$ if and only if there exists a real number r such that $r > 0$ and $\mathrm{Ball}(u,r) \subseteq P$.

(9) For all points u, v of $\mathcal{E}^2$ such that $u = [r_1,s_1]$ and $v = [r_2,s_2]$ holds $\rho(u,v) = \sqrt{(r_1-r_2)^2+(s_1-s_2)^2}$.

(10) For every point u of $\mathcal{E}^2$ such that $u = [r,s]$ holds if $0 \leq r_2$ and $r_2 < r_1$, then $[r+r_2,s] \in \mathrm{Ball}(u,r_1)$.

(11) For every point u of $\mathcal{E}^2$ such that $u = [r,s]$ holds if $0 \leq s_2$ and $s_2 < s_1$, then $[r,s+s_2] \in \mathrm{Ball}(u,s_1)$.

(12) For every point u of $\mathcal{E}^2$ such that $u = [r,s]$ holds if $0 \leq r_2$ and $r_2 < r_1$, then $[r-r_2,s] \in \mathrm{Ball}(u,r_1)$.

(13) For every point u of $\mathcal{E}^2$ such that $u = [r,s]$ holds if $0 \leq s_2$ and $s_2 < s_1$, then $[r,s-s_2] \in \mathrm{Ball}(u,s_1)$.

(14) If $1 \leq i$ and $i < \mathrm{len}\,G$ and $1 \leq j$ and $j < \mathrm{width}\,G$, then $(G \circ (i,j)) + (G \circ (i+1,j+1)) = (G \circ (i,j+1)) + (G \circ (i+1,j))$.

(15) $\mathrm{Int}\,\mathrm{vstrip}(G,0) = \{[r,s] : r < (G \circ (1,1))_{\mathbf{1}}\}$.

[1] The propositions (1)–(3) have been removed.
[2] The proposition (5) has been removed.

(16) $\operatorname{Int}\operatorname{vstrip}(G, \operatorname{len} G) = \{[r,s] : (G \circ (\operatorname{len} G, 1))_{\mathbf{1}} < r\}$.

(17) If $1 \leq i$ and $i < \operatorname{len} G$, then $\operatorname{Int}\operatorname{vstrip}(G,i) = \{[r,s] : (G \circ (i,1))_{\mathbf{1}} < r \wedge r < (G \circ (i+1,1))_{\mathbf{1}}\}$.

(18) $\operatorname{Int}\operatorname{hstrip}(G,0) = \{[r,s] : s < (G \circ (1,1))_{\mathbf{2}}\}$.

(19) $\operatorname{Int}\operatorname{hstrip}(G, \operatorname{width} G) = \{[r,s] : (G \circ (1, \operatorname{width} G))_{\mathbf{2}} < s\}$.

(20) If $1 \leq j$ and $j < \operatorname{width} G$, then $\operatorname{Int}\operatorname{hstrip}(G,j) = \{[r,s] : (G \circ (1,j))_{\mathbf{2}} < s \wedge s < (G \circ (1,j+1))_{\mathbf{2}}\}$.

(21) $\operatorname{Int}\operatorname{cell}(G,0,0) = \{[r,s] : r < (G \circ (1,1))_{\mathbf{1}} \wedge s < (G \circ (1,1))_{\mathbf{2}}\}$.

(22) $\operatorname{Int}\operatorname{cell}(G,0,\operatorname{width} G) = \{[r,s] : r < (G \circ (1,1))_{\mathbf{1}} \wedge (G \circ (1,\operatorname{width} G))_{\mathbf{2}} < s\}$.

(23) If $1 \leq j$ and $j < \operatorname{width} G$, then $\operatorname{Int}\operatorname{cell}(G,0,j) = \{[r,s] : r < (G \circ (1,1))_{\mathbf{1}} \wedge (G \circ (1,j))_{\mathbf{2}} < s \wedge s < (G \circ (1,j+1))_{\mathbf{2}}\}$.

(24) $\operatorname{Int}\operatorname{cell}(G,\operatorname{len} G,0) = \{[r,s] : (G \circ (\operatorname{len} G,1))_{\mathbf{1}} < r \wedge s < (G \circ (1,1))_{\mathbf{2}}\}$.

(25) $\operatorname{Int}\operatorname{cell}(G,\operatorname{len} G,\operatorname{width} G) = \{[r,s] : (G \circ (\operatorname{len} G,1))_{\mathbf{1}} < r \wedge (G \circ (1,\operatorname{width} G))_{\mathbf{2}} < s\}$.

(26) If $1 \leq j$ and $j < \operatorname{width} G$, then $\operatorname{Int}\operatorname{cell}(G,\operatorname{len} G,j) = \{[r,s] : (G \circ (\operatorname{len} G,1))_{\mathbf{1}} < r \wedge (G \circ (1,j))_{\mathbf{2}} < s \wedge s < (G \circ (1,j+1))_{\mathbf{2}}\}$.

(27) If $1 \leq i$ and $i < \operatorname{len} G$, then $\operatorname{Int}\operatorname{cell}(G,i,0) = \{[r,s] : (G \circ (i,1))_{\mathbf{1}} < r \wedge r < (G \circ (i+1,1))_{\mathbf{1}} \wedge s < (G \circ (1,1))_{\mathbf{2}}\}$.

(28) If $1 \leq i$ and $i < \operatorname{len} G$, then $\operatorname{Int}\operatorname{cell}(G,i,\operatorname{width} G) = \{[r,s] : (G \circ (i,1))_{\mathbf{1}} < r \wedge r < (G \circ (i+1,1))_{\mathbf{1}} \wedge (G \circ (1,\operatorname{width} G))_{\mathbf{2}} < s\}$.

(29) If $1 \leq i$ and $i < \operatorname{len} G$ and $1 \leq j$ and $j < \operatorname{width} G$, then $\operatorname{Int}\operatorname{cell}(G,i,j) = \{[r,s] : (G \circ (i,1))_{\mathbf{1}} < r \wedge r < (G \circ (i+1,1))_{\mathbf{1}} \wedge (G \circ (1,j))_{\mathbf{2}} < s \wedge s < (G \circ (1,j+1))_{\mathbf{2}}\}$.

(30) If $1 \leq j$ and $j \leq \operatorname{width} G$ and $p \in \operatorname{Int}\operatorname{hstrip}(G,j)$, then $p_{\mathbf{2}} > (G \circ (1,j))_{\mathbf{2}}$.

(31) If $j < \operatorname{width} G$ and $p \in \operatorname{Int}\operatorname{hstrip}(G,j)$, then $p_{\mathbf{2}} < (G \circ (1,j+1))_{\mathbf{2}}$.

(32) If $1 \leq i$ and $i \leq \operatorname{len} G$ and $p \in \operatorname{Int}\operatorname{vstrip}(G,i)$, then $p_{\mathbf{1}} > (G \circ (i,1))_{\mathbf{1}}$.

(33) If $i < \operatorname{len} G$ and $p \in \operatorname{Int}\operatorname{vstrip}(G,i)$, then $p_{\mathbf{1}} < (G \circ (i+1,1))_{\mathbf{1}}$.

(34) If $1 \leq i$ and $i+1 \leq \operatorname{len} G$ and $1 \leq j$ and $j+1 \leq \operatorname{width} G$, then $\frac{1}{2} \cdot ((G \circ (i,j)) + (G \circ (i+1,j+1))) \in \operatorname{Int}\operatorname{cell}(G,i,j)$.

(35) If $1 \leq i$ and $i+1 \leq \operatorname{len} G$, then $\frac{1}{2} \cdot ((G \circ (i,\operatorname{width} G)) + (G \circ (i+1,\operatorname{width} G))) + [0,1] \in \operatorname{Int}\operatorname{cell}(G,i,\operatorname{width} G)$.

(36) If $1 \leq i$ and $i+1 \leq \operatorname{len} G$, then $\frac{1}{2} \cdot ((G \circ (i,1)) + (G \circ (i+1,1))) - [0,1] \in \operatorname{Int}\operatorname{cell}(G,i,0)$.

(37) If $1 \leq j$ and $j+1 \leq \operatorname{width} G$, then $\frac{1}{2} \cdot ((G \circ (\operatorname{len} G,j)) + (G \circ (\operatorname{len} G,j+1))) + [1,0] \in \operatorname{Int}\operatorname{cell}(G,\operatorname{len} G,j)$.

(38) If $1 \leq j$ and $j+1 \leq \operatorname{width} G$, then $\frac{1}{2} \cdot ((G \circ (1,j)) + (G \circ (1,j+1))) - [1,0] \in \operatorname{Int}\operatorname{cell}(G,0,j)$.

(39) $(G \circ (1,1)) - [1,1] \in \operatorname{Int}\operatorname{cell}(G,0,0)$.

(40) $(G \circ (\operatorname{len} G,\operatorname{width} G)) + [1,1] \in \operatorname{Int}\operatorname{cell}(G,\operatorname{len} G,\operatorname{width} G)$.

(41) $(G \circ (1,\operatorname{width} G)) + [-1,1] \in \operatorname{Int}\operatorname{cell}(G,0,\operatorname{width} G)$.

(42) $(G \circ (\operatorname{len} G,1)) + [1,-1] \in \operatorname{Int}\operatorname{cell}(G,\operatorname{len} G,0)$.

(43) If $1 \leq i$ and $i < \operatorname{len} G$ and $1 \leq j$ and $j < \operatorname{width} G$, then $\mathcal{L}(\frac{1}{2} \cdot ((G \circ (i,j)) + (G \circ (i+1,j+1))), \frac{1}{2} \cdot ((G \circ (i,j)) + (G \circ (i,j+1)))) \subseteq \operatorname{Int}\operatorname{cell}(G,i,j) \cup \{\frac{1}{2} \cdot ((G \circ (i,j)) + (G \circ (i,j+1)))\}$.

(44) Suppose $1 \leq i$ and $i < \operatorname{len} G$ and $1 \leq j$ and $j < \operatorname{width} G$. Then $\mathcal{L}(\frac{1}{2} \cdot ((G \circ (i, j)) + (G \circ (i+1, j+1))), \frac{1}{2} \cdot ((G \circ (i, j+1)) + (G \circ (i+1, j+1)))) \subseteq \operatorname{Int} \operatorname{cell}(G, i, j) \cup \{\frac{1}{2} \cdot ((G \circ (i, j+1)) + (G \circ (i+1, j+1)))\}$.

(45) Suppose $1 \leq i$ and $i < \operatorname{len} G$ and $1 \leq j$ and $j < \operatorname{width} G$. Then $\mathcal{L}(\frac{1}{2} \cdot ((G \circ (i, j)) + (G \circ (i+1, j+1))), \frac{1}{2} \cdot ((G \circ (i+1, j)) + (G \circ (i+1, j+1)))) \subseteq \operatorname{Int} \operatorname{cell}(G, i, j) \cup \{\frac{1}{2} \cdot ((G \circ (i+1, j)) + (G \circ (i+1, j+1)))\}$.

(46) If $1 \leq i$ and $i < \operatorname{len} G$ and $1 \leq j$ and $j < \operatorname{width} G$, then $\mathcal{L}(\frac{1}{2} \cdot ((G \circ (i, j)) + (G \circ (i+1, j+1))), \frac{1}{2} \cdot ((G \circ (i, j)) + (G \circ (i+1, j)))) \subseteq \operatorname{Int} \operatorname{cell}(G, i, j) \cup \{\frac{1}{2} \cdot ((G \circ (i, j)) + (G \circ (i+1, j)))\}$.

(47) If $1 \leq j$ and $j < \operatorname{width} G$, then $\mathcal{L}(\frac{1}{2} \cdot ((G \circ (1, j)) + (G \circ (1, j+1))) - [1, 0], \frac{1}{2} \cdot ((G \circ (1, j)) + (G \circ (1, j+1)))) \subseteq \operatorname{Int} \operatorname{cell}(G, 0, j) \cup \{\frac{1}{2} \cdot ((G \circ (1, j)) + (G \circ (1, j+1)))\}$.

(48) If $1 \leq j$ and $j < \operatorname{width} G$, then $\mathcal{L}(\frac{1}{2} \cdot ((G \circ (\operatorname{len} G, j)) + (G \circ (\operatorname{len} G, j+1))) + [1, 0], \frac{1}{2} \cdot ((G \circ (\operatorname{len} G, j)) + (G \circ (\operatorname{len} G, j+1)))) \subseteq \operatorname{Int} \operatorname{cell}(G, \operatorname{len} G, j) \cup \{\frac{1}{2} \cdot ((G \circ (\operatorname{len} G, j)) + (G \circ (\operatorname{len} G, j+1)))\}$.

(49) If $1 \leq i$ and $i < \operatorname{len} G$, then $\mathcal{L}(\frac{1}{2} \cdot ((G \circ (i, 1)) + (G \circ (i+1, 1))) - [0, 1], \frac{1}{2} \cdot ((G \circ (i, 1)) + (G \circ (i+1, 1)))) \subseteq \operatorname{Int} \operatorname{cell}(G, i, 0) \cup \{\frac{1}{2} \cdot ((G \circ (i, 1)) + (G \circ (i+1, 1)))\}$.

(50) If $1 \leq i$ and $i < \operatorname{len} G$, then $\mathcal{L}(\frac{1}{2} \cdot ((G \circ (i, \operatorname{width} G)) + (G \circ (i+1, \operatorname{width} G))) + [0, 1], \frac{1}{2} \cdot ((G \circ (i, \operatorname{width} G)) + (G \circ (i+1, \operatorname{width} G)))) \subseteq \operatorname{Int} \operatorname{cell}(G, i, \operatorname{width} G) \cup \{\frac{1}{2} \cdot ((G \circ (i, \operatorname{width} G)) + (G \circ (i+1, \operatorname{width} G)))\}$.

(51) If $1 \leq j$ and $j < \operatorname{width} G$, then $\mathcal{L}(\frac{1}{2} \cdot ((G \circ (1, j)) + (G \circ (1, j+1))) - [1, 0], (G \circ (1, j)) - [1, 0]) \subseteq \operatorname{Int} \operatorname{cell}(G, 0, j) \cup \{(G \circ (1, j)) - [1, 0]\}$.

(52) If $1 \leq j$ and $j < \operatorname{width} G$, then $\mathcal{L}(\frac{1}{2} \cdot ((G \circ (1, j)) + (G \circ (1, j+1))) - [1, 0], (G \circ (1, j+1)) - [1, 0]) \subseteq \operatorname{Int} \operatorname{cell}(G, 0, j) \cup \{(G \circ (1, j+1)) - [1, 0]\}$.

(53) If $1 \leq j$ and $j < \operatorname{width} G$, then $\mathcal{L}(\frac{1}{2} \cdot ((G \circ (\operatorname{len} G, j)) + (G \circ (\operatorname{len} G, j+1))) + [1, 0], (G \circ (\operatorname{len} G, j)) + [1, 0]) \subseteq \operatorname{Int} \operatorname{cell}(G, \operatorname{len} G, j) \cup \{(G \circ (\operatorname{len} G, j)) + [1, 0]\}$.

(54) If $1 \leq j$ and $j < \operatorname{width} G$, then $\mathcal{L}(\frac{1}{2} \cdot ((G \circ (\operatorname{len} G, j)) + (G \circ (\operatorname{len} G, j+1))) + [1, 0], (G \circ (\operatorname{len} G, j+1)) + [1, 0]) \subseteq \operatorname{Int} \operatorname{cell}(G, \operatorname{len} G, j) \cup \{(G \circ (\operatorname{len} G, j+1)) + [1, 0]\}$.

(55) If $1 \leq i$ and $i < \operatorname{len} G$, then $\mathcal{L}(\frac{1}{2} \cdot ((G \circ (i, 1)) + (G \circ (i+1, 1))) - [0, 1], (G \circ (i, 1)) - [0, 1]) \subseteq \operatorname{Int} \operatorname{cell}(G, i, 0) \cup \{(G \circ (i, 1)) - [0, 1]\}$.

(56) If $1 \leq i$ and $i < \operatorname{len} G$, then $\mathcal{L}(\frac{1}{2} \cdot ((G \circ (i, 1)) + (G \circ (i+1, 1))) - [0, 1], (G \circ (i+1, 1)) - [0, 1]) \subseteq \operatorname{Int} \operatorname{cell}(G, i, 0) \cup \{(G \circ (i+1, 1)) - [0, 1]\}$.

(57) If $1 \leq i$ and $i < \operatorname{len} G$, then $\mathcal{L}(\frac{1}{2} \cdot ((G \circ (i, \operatorname{width} G)) + (G \circ (i+1, \operatorname{width} G))) + [0, 1], (G \circ (i, \operatorname{width} G)) + [0, 1]) \subseteq \operatorname{Int} \operatorname{cell}(G, i, \operatorname{width} G) \cup \{(G \circ (i, \operatorname{width} G)) + [0, 1]\}$.

(58) If $1 \leq i$ and $i < \operatorname{len} G$, then $\mathcal{L}(\frac{1}{2} \cdot ((G \circ (i, \operatorname{width} G)) + (G \circ (i+1, \operatorname{width} G))) + [0, 1], (G \circ (i+1, \operatorname{width} G)) + [0, 1]) \subseteq \operatorname{Int} \operatorname{cell}(G, i, \operatorname{width} G) \cup \{(G \circ (i+1, \operatorname{width} G)) + [0, 1]\}$.

(59) $\mathcal{L}((G \circ (1, 1)) - [1, 1], (G \circ (1, 1)) - [1, 0]) \subseteq \operatorname{Int} \operatorname{cell}(G, 0, 0) \cup \{(G \circ (1, 1)) - [1, 0]\}$.

(60) $\mathcal{L}((G \circ (\operatorname{len} G, 1)) + [1, -1], (G \circ (\operatorname{len} G, 1)) + [1, 0]) \subseteq \operatorname{Int} \operatorname{cell}(G, \operatorname{len} G, 0) \cup \{(G \circ (\operatorname{len} G, 1)) + [1, 0]\}$.

(61) $\mathcal{L}((G \circ (1, \operatorname{width} G)) + [-1, 1], (G \circ (1, \operatorname{width} G)) - [1, 0]) \subseteq \operatorname{Int} \operatorname{cell}(G, 0, \operatorname{width} G) \cup \{(G \circ (1, \operatorname{width} G)) - [1, 0]\}$.

(62) $\mathcal{L}((G \circ (\operatorname{len} G, \operatorname{width} G)) + [1, 1], (G \circ (\operatorname{len} G, \operatorname{width} G)) + [1, 0]) \subseteq \operatorname{Int} \operatorname{cell}(G, \operatorname{len} G, \operatorname{width} G) \cup \{(G \circ (\operatorname{len} G, \operatorname{width} G)) + [1, 0]\}$.

(63) $\mathcal{L}((G \circ (1, 1)) - [1, 1], (G \circ (1, 1)) - [0, 1]) \subseteq \operatorname{Int} \operatorname{cell}(G, 0, 0) \cup \{(G \circ (1, 1)) - [0, 1]\}$.

(64) $\mathcal{L}((G\circ(\operatorname{len}G,1))+[1,-1],(G\circ(\operatorname{len}G,1))-[0,1])\subseteq \operatorname{Int}\operatorname{cell}(G,\operatorname{len}G,0)\cup\{(G\circ(\operatorname{len}G,1))-[0,1]\}$.

(65) $\mathcal{L}((G\circ(1,\operatorname{width}G))+[-1,1],(G\circ(1,\operatorname{width}G))+[0,1])\subseteq \operatorname{Int}\operatorname{cell}(G,0,\operatorname{width}G)\cup\{(G\circ(1,\operatorname{width}G))+[0,1]\}$.

(66) $\mathcal{L}((G\circ(\operatorname{len}G,\operatorname{width}G))+[1,1],(G\circ(\operatorname{len}G,\operatorname{width}G))+[0,1])\subseteq \operatorname{Int}\operatorname{cell}(G,\operatorname{len}G,\operatorname{width}G)\cup\{(G\circ(\operatorname{len}G,\operatorname{width}G))+[0,1]\}$.

(67) Suppose $1\leq i$ and $i<\operatorname{len}G$ and $1\leq j$ and $j+1<\operatorname{width}G$. Then $\mathcal{L}(\frac{1}{2}\cdot((G\circ(i,j))+(G\circ(i+1,j+1))),\frac{1}{2}\cdot((G\circ(i,j+1))+(G\circ(i+1,j+2))))\subseteq \operatorname{Int}\operatorname{cell}(G,i,j)\cup\operatorname{Int}\operatorname{cell}(G,i,j+1)\cup\{\frac{1}{2}\cdot((G\circ(i,j+1))+(G\circ(i+1,j+1)))\}$.

(68) Suppose $1\leq j$ and $j<\operatorname{width}G$ and $1\leq i$ and $i+1<\operatorname{len}G$. Then $\mathcal{L}(\frac{1}{2}\cdot((G\circ(i,j))+(G\circ(i+1,j+1))),\frac{1}{2}\cdot((G\circ(i+1,j))+(G\circ(i+2,j+1))))\subseteq \operatorname{Int}\operatorname{cell}(G,i,j)\cup\operatorname{Int}\operatorname{cell}(G,i+1,j)\cup\{\frac{1}{2}\cdot((G\circ(i+1,j))+(G\circ(i+1,j+1)))\}$.

(69) If $1\leq i$ and $i<\operatorname{len}G$ and $1<\operatorname{width}G$, then $\mathcal{L}(\frac{1}{2}\cdot((G\circ(i,1))+(G\circ(i+1,1)))-[0,1],\frac{1}{2}\cdot((G\circ(i,1))+(G\circ(i+1,2))))\subseteq \operatorname{Int}\operatorname{cell}(G,i,0)\cup\operatorname{Int}\operatorname{cell}(G,i,1)\cup\{\frac{1}{2}\cdot((G\circ(i,1))+(G\circ(i+1,1)))\}$.

(70) Suppose $1\leq i$ and $i<\operatorname{len}G$ and $1<\operatorname{width}G$. Then $\mathcal{L}(\frac{1}{2}\cdot((G\circ(i,\operatorname{width}G))+(G\circ(i+1,\operatorname{width}G)))+[0,1],\frac{1}{2}\cdot((G\circ(i,\operatorname{width}G))+(G\circ(i+1,\operatorname{width}G-'1))))\subseteq \operatorname{Int}\operatorname{cell}(G,i,\operatorname{width}G-'1)\cup\operatorname{Int}\operatorname{cell}(G,i,\operatorname{width}G)\cup\{\frac{1}{2}\cdot((G\circ(i,\operatorname{width}G))+(G\circ(i+1,\operatorname{width}G)))\}$.

(71) If $1\leq j$ and $j<\operatorname{width}G$ and $1<\operatorname{len}G$, then $\mathcal{L}(\frac{1}{2}\cdot((G\circ(1,j))+(G\circ(1,j+1)))-[1,0],\frac{1}{2}\cdot((G\circ(1,j))+(G\circ(2,j+1))))\subseteq \operatorname{Int}\operatorname{cell}(G,0,j)\cup\operatorname{Int}\operatorname{cell}(G,1,j)\cup\{\frac{1}{2}\cdot((G\circ(1,j))+(G\circ(1,j+1)))\}$.

(72) Suppose $1\leq j$ and $j<\operatorname{width}G$ and $1<\operatorname{len}G$. Then $\mathcal{L}(\frac{1}{2}\cdot((G\circ(\operatorname{len}G,j))+(G\circ(\operatorname{len}G,j+1)))+[1,0],\frac{1}{2}\cdot((G\circ(\operatorname{len}G,j))+(G\circ(\operatorname{len}G-'1,j+1))))\subseteq \operatorname{Int}\operatorname{cell}(G,\operatorname{len}G-'1,j)\cup\operatorname{Int}\operatorname{cell}(G,\operatorname{len}G,j)\cup\{\frac{1}{2}\cdot((G\circ(\operatorname{len}G,j))+(G\circ(\operatorname{len}G,j+1)))\}$.

(73) If $1<\operatorname{len}G$ and $1\leq j$ and $j+1<\operatorname{width}G$, then $\mathcal{L}(\frac{1}{2}\cdot((G\circ(1,j))+(G\circ(1,j+1)))-[1,0],\frac{1}{2}\cdot((G\circ(1,j+1))+(G\circ(1,j+2)))-[1,0])\subseteq \operatorname{Int}\operatorname{cell}(G,0,j)\cup\operatorname{Int}\operatorname{cell}(G,0,j+1)\cup\{(G\circ(1,j+1))-[1,0]\}$.

(74) Suppose $1<\operatorname{len}G$ and $1\leq j$ and $j+1<\operatorname{width}G$. Then $\mathcal{L}(\frac{1}{2}\cdot((G\circ(\operatorname{len}G,j))+(G\circ(\operatorname{len}G,j+1)))+[1,0],\frac{1}{2}\cdot((G\circ(\operatorname{len}G,j+1))+(G\circ(\operatorname{len}G,j+2)))+[1,0])\subseteq \operatorname{Int}\operatorname{cell}(G,\operatorname{len}G,j)\cup\operatorname{Int}\operatorname{cell}(G,\operatorname{len}G,j+1)\cup\{(G\circ(\operatorname{len}G,j+1))+[1,0]\}$.

(75) If $1<\operatorname{width}G$ and $1\leq i$ and $i+1<\operatorname{len}G$, then $\mathcal{L}(\frac{1}{2}\cdot((G\circ(i,1))+(G\circ(i+1,1)))-[0,1],\frac{1}{2}\cdot((G\circ(i+1,1))+(G\circ(i+2,1)))-[0,1])\subseteq \operatorname{Int}\operatorname{cell}(G,i,0)\cup\operatorname{Int}\operatorname{cell}(G,i+1,0)\cup\{(G\circ(i+1,1))-[0,1]\}$.

(76) Suppose $1<\operatorname{width}G$ and $1\leq i$ and $i+1<\operatorname{len}G$. Then $\mathcal{L}(\frac{1}{2}\cdot((G\circ(i,\operatorname{width}G))+(G\circ(i+1,\operatorname{width}G)))+[0,1],\frac{1}{2}\cdot((G\circ(i+1,\operatorname{width}G))+(G\circ(i+2,\operatorname{width}G)))+[0,1])\subseteq \operatorname{Int}\operatorname{cell}(G,i,\operatorname{width}G)\cup\operatorname{Int}\operatorname{cell}(G,i+1,\operatorname{width}G)\cup\{(G\circ(i+1,\operatorname{width}G))+[0,1]\}$.

(77) If $1<\operatorname{len}G$ and $1<\operatorname{width}G$, then $\mathcal{L}((G\circ(1,1))-[1,1],\frac{1}{2}\cdot((G\circ(1,1))+(G\circ(1,2)))-[1,0])\subseteq \operatorname{Int}\operatorname{cell}(G,0,0)\cup\operatorname{Int}\operatorname{cell}(G,0,1)\cup\{(G\circ(1,1))-[1,0]\}$.

(78) If $1<\operatorname{len}G$ and $1<\operatorname{width}G$, then $\mathcal{L}((G\circ(\operatorname{len}G,1))+[1,-1],\frac{1}{2}\cdot((G\circ(\operatorname{len}G,1))+(G\circ(\operatorname{len}G,2)))+[1,0])\subseteq \operatorname{Int}\operatorname{cell}(G,\operatorname{len}G,0)\cup\operatorname{Int}\operatorname{cell}(G,\operatorname{len}G,1)\cup\{(G\circ(\operatorname{len}G,1))+[1,0]\}$.

(79) If $1<\operatorname{len}G$ and $1<\operatorname{width}G$, then $\mathcal{L}((G\circ(1,\operatorname{width}G))+[-1,1],\frac{1}{2}\cdot((G\circ(1,\operatorname{width}G))+(G\circ(1,\operatorname{width}G-'1)))-[1,0])\subseteq \operatorname{Int}\operatorname{cell}(G,0,\operatorname{width}G)\cup\operatorname{Int}\operatorname{cell}(G,0,\operatorname{width}G-'1)\cup\{(G\circ(1,\operatorname{width}G))-[1,0]\}$.

(80) If $1 < \mathrm{len}\,G$ and $1 < \mathrm{width}\,G$, then $\mathcal{L}((G \circ (\mathrm{len}\,G, \mathrm{width}\,G)) + [1,1], \frac{1}{2} \cdot ((G \circ (\mathrm{len}\,G, \mathrm{width}\,G)) + (G \circ (\mathrm{len}\,G, \mathrm{width}\,G -' 1))) + [1,0]) \subseteq \mathrm{Int}\,\mathrm{cell}(G, \mathrm{len}\,G, \mathrm{width}\,G) \cup \mathrm{Int}\,\mathrm{cell}(G, \mathrm{len}\,G, \mathrm{width}\,G -' 1) \cup \{(G \circ (\mathrm{len}\,G, \mathrm{width}\,G)) + [1,0]\}$.

(81) If $1 < \mathrm{width}\,G$ and $1 < \mathrm{len}\,G$, then $\mathcal{L}((G \circ (1,1)) - [1,1], \frac{1}{2} \cdot ((G \circ (1,1)) + (G \circ (2,1))) - [0,1]) \subseteq \mathrm{Int}\,\mathrm{cell}(G,0,0) \cup \mathrm{Int}\,\mathrm{cell}(G,1,0) \cup \{(G \circ (1,1)) - [0,1]\}$.

(82) If $1 < \mathrm{width}\,G$ and $1 < \mathrm{len}\,G$, then $\mathcal{L}((G \circ (1, \mathrm{width}\,G)) + [-1,1], \frac{1}{2} \cdot ((G \circ (1, \mathrm{width}\,G)) + (G \circ (2, \mathrm{width}\,G))) + [0,1]) \subseteq \mathrm{Int}\,\mathrm{cell}(G, 0, \mathrm{width}\,G) \cup \mathrm{Int}\,\mathrm{cell}(G, 1, \mathrm{width}\,G) \cup \{(G \circ (1, \mathrm{width}\,G)) + [0,1]\}$.

(83) If $1 < \mathrm{width}\,G$ and $1 < \mathrm{len}\,G$, then $\mathcal{L}((G \circ (\mathrm{len}\,G, 1)) + [1,-1], \frac{1}{2} \cdot ((G \circ (\mathrm{len}\,G, 1)) + (G \circ (\mathrm{len}\,G -' 1, 1))) - [0,1]) \subseteq \mathrm{Int}\,\mathrm{cell}(G, \mathrm{len}\,G, 0) \cup \mathrm{Int}\,\mathrm{cell}(G, \mathrm{len}\,G -' 1, 0) \cup \{(G \circ (\mathrm{len}\,G, 1)) - [0,1]\}$.

(84) If $1 < \mathrm{width}\,G$ and $1 < \mathrm{len}\,G$, then $\mathcal{L}((G \circ (\mathrm{len}\,G, \mathrm{width}\,G)) + [1,1], \frac{1}{2} \cdot ((G \circ (\mathrm{len}\,G, \mathrm{width}\,G)) + (G \circ (\mathrm{len}\,G -' 1, \mathrm{width}\,G))) + [0,1]) \subseteq \mathrm{Int}\,\mathrm{cell}(G, \mathrm{len}\,G, \mathrm{width}\,G) \cup \mathrm{Int}\,\mathrm{cell}(G, \mathrm{len}\,G -' 1, \mathrm{width}\,G) \cup \{(G \circ (\mathrm{len}\,G, \mathrm{width}\,G)) + [0,1]\}$.

(85) If $1 \leq i$ and $i+1 \leq \mathrm{len}\,G$ and $1 \leq j$ and $j+1 \leq \mathrm{width}\,G$, then $\mathcal{L}(\frac{1}{2} \cdot ((G \circ (i,j)) + (G \circ (i+1, j+1))), p)$ meets $\mathrm{Int}\,\mathrm{cell}(G,i,j)$.

(86) If $1 \leq i$ and $i+1 \leq \mathrm{len}\,G$, then $\mathcal{L}(p, \frac{1}{2} \cdot ((G \circ (i, \mathrm{width}\,G)) + (G \circ (i+1, \mathrm{width}\,G))) + [0,1])$ meets $\mathrm{Int}\,\mathrm{cell}(G,i,\mathrm{width}\,G)$.

(87) If $1 \leq i$ and $i+1 \leq \mathrm{len}\,G$, then $\mathcal{L}(\frac{1}{2} \cdot ((G \circ (i,1)) + (G \circ (i+1,1))) - [0,1], p)$ meets $\mathrm{Int}\,\mathrm{cell}(G,i,0)$.

(88) If $1 \leq j$ and $j+1 \leq \mathrm{width}\,G$, then $\mathcal{L}(\frac{1}{2} \cdot ((G \circ (1,j)) + (G \circ (1,j+1))) - [1,0], p)$ meets $\mathrm{Int}\,\mathrm{cell}(G,0,j)$.

(89) If $1 \leq j$ and $j+1 \leq \mathrm{width}\,G$, then $\mathcal{L}(p, \frac{1}{2} \cdot ((G \circ (\mathrm{len}\,G, j)) + (G \circ (\mathrm{len}\,G, j+1))) + [1,0])$ meets $\mathrm{Int}\,\mathrm{cell}(G, \mathrm{len}\,G, j)$.

(90) $\mathcal{L}(p, (G \circ (1,1)) - [1,1])$ meets $\mathrm{Int}\,\mathrm{cell}(G,0,0)$.

(91) $\mathcal{L}(p, (G \circ (\mathrm{len}\,G, \mathrm{width}\,G)) + [1,1])$ meets $\mathrm{Int}\,\mathrm{cell}(G, \mathrm{len}\,G, \mathrm{width}\,G)$.

(92) $\mathcal{L}(p, (G \circ (1, \mathrm{width}\,G)) + [-1,1])$ meets $\mathrm{Int}\,\mathrm{cell}(G, 0, \mathrm{width}\,G)$.

(93) $\mathcal{L}(p, (G \circ (\mathrm{len}\,G, 1)) + [1,-1])$ meets $\mathrm{Int}\,\mathrm{cell}(G, \mathrm{len}\,G, 0)$.

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