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Go-Board Theorem

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Summary. We prove the Go-board theorem which is a special case of Hex Theorem. The article is based on [11].

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The articles [12], [4], [15], [13], [1], [3], [2], [10], [14], [7], [5], [6], [9], and [8] provide the notation and terminology for this paper.

For simplicity, we use the following convention: p, p_1, p_2, q_1, q_2 denote points of $\mathcal{E}^2_{\rm T}$, P_1, P_2 denote subsets of $\mathcal{E}^2_{\rm T}$, f, f_1, f_2 denote finite sequences of elements of $\mathcal{E}^2_{\rm T}$, n denotes a natural number, and G denotes a Go-board.

One can prove the following propositions:

(1) Let given G, f_1, f_2. Suppose that $1 \leq \operatorname{len} f_1$ and $1 \leq \operatorname{len} f_2$ and f_1 is a sequence which elements belong to G and f_2 is a sequence which elements belong to G and $(f_1)_1 \in \operatorname{rng} \operatorname{Line}(G, 1)$ and $(f_1)_{\operatorname{len} f_1} \in \operatorname{rng} \operatorname{Line}(G, \operatorname{len} G)$ and $(f_2)_1 \in \operatorname{rng}(G_{\square,1})$ and $(f_2)_{\operatorname{len} f_2} \in \operatorname{rng}(G_{\square,\operatorname{width} G})$. Then $\operatorname{rng} f_1$ meets $\operatorname{rng} f_2$.

(2) Let given G, f_1, f_2. Suppose that $2 \leq \operatorname{len} f_1$ and $2 \leq \operatorname{len} f_2$ and f_1 is a sequence which elements belong to G and f_2 is a sequence which elements belong to G and $(f_1)_1 \in \operatorname{rng} \operatorname{Line}(G, 1)$ and $(f_1)_{\operatorname{len} f_1} \in \operatorname{rng} \operatorname{Line}(G, \operatorname{len} G)$ and $(f_2)_1 \in \operatorname{rng}(G_{\square,1})$ and $(f_2)_{\operatorname{len} f_2} \in \operatorname{rng}(G_{\square,\operatorname{width} G})$. Then $\widetilde{\mathcal{L}}(f_1)$ meets $\widetilde{\mathcal{L}}(f_2)$.

(3) Let given G, f_1, f_2. Suppose that $2 \leq \operatorname{len} f_1$ and $2 \leq \operatorname{len} f_2$ and f_1 is a sequence which elements belong to G and f_2 is a sequence which elements belong to G and $(f_1)_1 \in \operatorname{rng} \operatorname{Line}(G, 1)$ and $(f_1)_{\operatorname{len} f_1} \in \operatorname{rng} \operatorname{Line}(G, \operatorname{len} G)$ and $(f_2)_1 \in \operatorname{rng}(G_{\square,1})$ and $(f_2)_{\operatorname{len} f_2} \in \operatorname{rng}(G_{\square,\operatorname{width} G})$. Then $\widetilde{\mathcal{L}}(f_1)$ meets $\widetilde{\mathcal{L}}(f_2)$.

Let f be a finite sequence of elements of $\mathbb{R}$ and let r, s be real numbers. We say that f lies between r and s if and only if:

(Def. 1) For every n such that $n \in \operatorname{dom} f$ holds $r \leq f(n)$ and $f(n) \leq s$.

Let D be a non empty set, let f_1 be a finite sequence of elements of D, and let f_2 be a non empty finite sequence of elements of D. Note that $f_1 \frown f_2$ is non empty and $f_2 \frown f_1$ is non empty.

Next we state three propositions:

(4) Let f_1, f_2 be finite sequences of elements of $\mathcal{E}^2_{\rm T}$. Suppose that $2 \leq \operatorname{len} f_1$ and $2 \leq \operatorname{len} f_2$ and f_1 is special and f_2 is special and for every n such that $n \in \operatorname{dom} f_1$ and $n+1 \in \operatorname{dom} f_1$ holds $(f_1)_n \neq (f_1)_{n+1}$ and for every n such that $n \in \operatorname{dom} f_2$ and $n+1 \in \operatorname{dom} f_2$ holds $(f_2)_n \neq (f_2)_{n+1}$ and $\mathbf{X}$-coordinate(f_1) lies between $(\mathbf{X}\text{-coordinate}(f_1))(1)$ and $(\mathbf{X}\text{-coordinate}(f_1))(\operatorname{len} f_1)$

and **X**-coordinate(f_2) lies between $(\mathbf{X}\text{-coordinate}(f_1))(1)$ and $(\mathbf{X}\text{-coordinate}(f_1))(\operatorname{len} f_1)$ and **Y**-coordinate(f_1) lies between $(\mathbf{Y}\text{-coordinate}(f_2))(1)$ and $(\mathbf{Y}\text{-coordinate}(f_2))(\operatorname{len} f_2)$ and **Y**-coordinate(f_2) lies between $(\mathbf{Y}\text{-coordinate}(f_2))(1)$ and $(\mathbf{Y}\text{-coordinate}(f_2))(\operatorname{len} f_2)$. Then $\widetilde{\mathcal{L}}(f_1)$ meets $\widetilde{\mathcal{L}}(f_2)$.

(5) Let f_1, f_2 be finite sequences of elements of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that f_1 is one-to-one and special and $2 \leq \operatorname{len} f_1$ and f_2 is one-to-one and special and $2 \leq \operatorname{len} f_2$ and **X**-coordinate(f_1) lies between $(\mathbf{X}\text{-coordinate}(f_1))(1)$ and $(\mathbf{X}\text{-coordinate}(f_1))(\operatorname{len} f_1)$ and **X**-coordinate(f_2) lies between $(\mathbf{X}\text{-coordinate}(f_1))(1)$ and $(\mathbf{X}\text{-coordinate}(f_1))(\operatorname{len} f_1)$ and **Y**-coordinate(f_1) lies between $(\mathbf{Y}\text{-coordinate}(f_2))(1)$ and $(\mathbf{Y}\text{-coordinate}(f_2))(\operatorname{len} f_2)$ and **Y**-coordinate(f_2) lies between $(\mathbf{Y}\text{-coordinate}(f_2))(1)$ and $(\mathbf{Y}\text{-coordinate}(f_2))(\operatorname{len} f_2)$. Then $\widetilde{\mathcal{L}}(f_1)$ meets $\widetilde{\mathcal{L}}(f_2)$.

(8)[1] Let given P_1, P_2, p_1, p_2, q_1, q_2. Suppose that

(i) P_1 is a special polygonal arc joining p_1 and q_1,

(ii) P_2 is a special polygonal arc joining p_2 and q_2,

(iii) for every p such that $p \in P_1 \cup P_2$ holds $(p_1)_{\mathbf{1}} \leq p_{\mathbf{1}}$ and $p_{\mathbf{1}} \leq (q_1)_{\mathbf{1}}$, and

(iv) for every p such that $p \in P_1 \cup P_2$ holds $(p_2)_{\mathbf{2}} \leq p_{\mathbf{2}}$ and $p_{\mathbf{2}} \leq (q_2)_{\mathbf{2}}$.

Then P_1 meets P_2.

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