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Inst. of Computer Science, Univ. of Białystok

Finite Sets

Agata Darmochwał
Warsaw University
Białystok

Summary. The article contains the definition of a finite set based on the notion of finite sequence. Some theorems about properties of finite sets and finite families of sets are proved.

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The articles [6], [4], [7], [8], [3], [5], [1], and [2] provide the notation and terminology for this paper.

Let I_1 be a set. We say that I_1 is finite if and only if:

(Def. 1) There exists a function p such that $\operatorname{rng} p = I_1$ and $\operatorname{dom} p \in \omega$.

We introduce I_1 is infinite as an antonym of I_1 is finite.

We adopt the following convention: A, B, C, D, X, Y are sets and f is a function.

Let us mention that there exists a set which is non empty and finite.

One can verify that every set which is empty is also finite.

Let X be a set. Observe that there exists a subset of X which is empty and finite.

The scheme *OLambdaC* deals with a set $\mathcal{A}$, a unary functor $\mathcal{F}$ yielding a set, a unary functor $\mathcal{G}$ yielding a set, and a unary predicate $\mathcal{P}$, and states that:

> There exists a function f such that $\operatorname{dom} f = \mathcal{A}$ and for every ordinal number x such that $x \in \mathcal{A}$ holds if $\mathcal{P}[x]$, then $f(x) = \mathcal{F}(x)$ and if not $\mathcal{P}[x]$, then $f(x) = \mathcal{G}(x)$

for all values of the parameters.

Let x be a set. One can verify that $\{x\}$ is finite.

Let X be a non empty set. Observe that there exists a subset of X which is non empty and finite.

Let x, y be sets. One can verify that $\{x,y\}$ is finite.

Let x, y, z be sets. One can check that $\{x,y,z\}$ is finite.

Let x_1, x_2, x_3, x_4 be sets. One can verify that $\{x_1,x_2,x_3,x_4\}$ is finite.

Let x_1, x_2, x_3, x_4, x_5 be sets. Observe that $\{x_1,x_2,x_3,x_4,x_5\}$ is finite.

Let x_1, x_2, x_3, x_4, x_5, x_6 be sets. Observe that $\{x_1,x_2,x_3,x_4,x_5,x_6\}$ is finite.

Let x_1, x_2, x_3, x_4, x_5, x_6, x_7 be sets. Observe that $\{x_1,x_2,x_3,x_4,x_5,x_6,x_7\}$ is finite.

Let x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 be sets. One can verify that $\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8\}$ is finite.

Let B be a finite set. Note that every subset of B is finite.

Let X, Y be finite sets. Note that $X \cup Y$ is finite.

One can prove the following two propositions:

(13)[1] If $A \subseteq B$ and B is finite, then A is finite.

(14) If A is finite and B is finite, then $A \cup B$ is finite.

[1] The propositions (1)–(12) have been removed.

Let A be a set and let B be a finite set. One can check that $A \cap B$ is finite.
Let A be a finite set and let B be a set. Observe that $A \cap B$ is finite and $A \setminus B$ is finite.
Next we state three propositions:

(15) If A is finite, then $A \cap B$ is finite.

(16) If A is finite, then $A \setminus B$ is finite.

(17) If A is finite, then $f^\circ A$ is finite.

Let f be a function and let A be a finite set. Observe that $f^\circ A$ is finite.
One can prove the following proposition

(18) Suppose A is finite. Let X be a family of subsets of A. Suppose $X \neq \emptyset$. Then there exists a set x such that $x \in X$ and for every set B such that $B \in X$ holds if $x \subseteq B$, then $B = x$.

The scheme *Finite* deals with a set $\mathcal{A}$ and a unary predicate $\mathcal{P}$, and states that:
$\mathcal{P}[\mathcal{A}]$
provided the following requirements are met:

- $\mathcal{A}$ is finite,
- $\mathcal{P}[\emptyset]$, and
- For all sets x, B such that $x \in \mathcal{A}$ and $B \subseteq \mathcal{A}$ and $\mathcal{P}[B]$ holds $\mathcal{P}[B \cup \{x\}]$.

One can prove the following proposition

(19) If A is finite and B is finite, then $[:A, B:]$ is finite.

Let A, B be finite sets. Note that $[:A, B:]$ is finite.
We now state the proposition

(20) If A is finite and B is finite and C is finite, then $[:A, B, C:]$ is finite.

Let A, B, C be finite sets. One can check that $[:A, B, C:]$ is finite.
Next we state the proposition

(21) If A is finite and B is finite and C is finite and D is finite, then $[:A, B, C, D:]$ is finite.

Let A, B, C, D be finite sets. Observe that $[:A, B, C, D:]$ is finite.
The following propositions are true:

(22) If $B \neq \emptyset$ and $[:A, B:]$ is finite, then A is finite.

(23) If $A \neq \emptyset$ and $[:A, B:]$ is finite, then B is finite.

(24) A is finite iff 2^A is finite.

(25) A is finite and for every X such that $X \in A$ holds X is finite iff $\bigcup A$ is finite.

(26) If $\operatorname{dom} f$ is finite, then $\operatorname{rng} f$ is finite.

(27) If $Y \subseteq \operatorname{rng} f$ and $f^{-1}(Y)$ is finite, then Y is finite.

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