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Non-Negative Real Numbers. Part II[1]

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The articles [2], [4], [1], and [3] provide the notation and terminology for this paper.

In this paper x, y, z denote elements of $\mathbb{R}_+$.

Next we state several propositions:

(1) If $x+y=y$, then $x=\emptyset$.

(2) If $x*y=\emptyset$, then $x=\emptyset$ or $y=\emptyset$.

(3) If $x \leq y$ and $y \leq z$, then $x \leq z$.

(4) If $x \leq y$ and $y \leq x$, then $x=y$.

(5) If $x \leq y$ and $y=\emptyset$, then $x=\emptyset$.

(6) If $x=\emptyset$, then $x \leq y$.

(7) $x \leq y$ iff $x+z \leq y+z$.

(8) If $x \leq y$, then $x*z \leq y*z$.

Let x, y be elements of $\mathbb{R}_+$. The functor $x -' y$ yields an element of $\mathbb{R}_+$ and is defined as follows:

(Def. 1)(i) $(x -' y)+y=x$ if $y \leq x$,

(ii) $x -' y=\emptyset$, otherwise.

The following propositions are true:

(9) $x \leq y$ or $x -' y \neq \emptyset$.

(10) If $x \leq y$ and $y -' x=\emptyset$, then $x=y$.

(11) $x -' y \leq x$.

(12) If $y \leq x$ and $y \leq z$, then $x+(z -' y)=(x -' y)+z$.

(13) If $z \leq y$, then $x+(y -' z)=(x+y) -' z$.

(14) If $z \leq x$ and $y \leq z$, then $(x -' z)+y=x -' (z -' y)$.

(15) If $y \leq x$ and $y \leq z$, then $(z -' y)+x=(x -' y)+z$.

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(16) If $x \leq y$, then $z -' y \leq z -' x$.

(17) If $x \leq y$, then $x -' z \leq y -' z$.

Let x, y be elements of $\mathbb{R}_+$. The functor $x - y$ is defined as follows:

(Def. 2) $$x - y = \begin{cases} \text{(i)} \quad x -' y, & \text{if } y \leq x, \\ \langle \emptyset, y -' x \rangle, & \text{otherwise.} \end{cases}$$

The following propositions are true:

(18) $x - x = \emptyset$.

(19) If $x = \emptyset$ and $y \neq \emptyset$, then $x - y = \langle \emptyset, y \rangle$.

(20) If $z \leq y$, then $x + (y -' z) = (x + y) - z$.

(21) If $z \not\leq y$, then $x - (z -' y) = (x + y) - z$.

(22) If $y \leq x$ and $y \not\leq z$, then $x - (y -' z) = (x -' y) + z$.

(23) If $y \not\leq x$ and $y \not\leq z$, then $x - (y -' z) = z - (y -' x)$.

(24) If $y \leq x$, then $x - (y + z) = (x -' y) - z$.

(25) If $x \leq y$ and $z \leq y$, then $(y -' z) - x = (y -' x) - z$.

(26) If $z \leq y$, then $x * (y -' z) = x * y - x * z$.

(27) If $z \not\leq y$ and $x \neq \emptyset$, then $\langle \emptyset, x * (z -' y) \rangle = x * y - x * z$.

(28) If $y -' z \neq \emptyset$ and $z \leq y$ and $x \neq \emptyset$, then $x * z - x * y = \langle \emptyset, x * (y -' z) \rangle$.

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