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Classical Configurations in Affine Planes[1]

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Summary. The classical sequence of implications which hold between Desargues and Pappus Axioms is proved. Formally Minor and Major Desargues Axiom (as suitable properties – predicates – of an affine plane) together with all its indirect forms are introduced; the same procedure is applied to Pappus Axioms. The so called Trapezium Desargues Axiom is also considered.

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The articles [1], [2], and [3] provide the notation and terminology for this paper.

We adopt the following rules: A_1 is an affine plane, a, a', b, b', c, c', o are elements of A_1, and A, C, K, M, N, P are subsets of A_1.

Let us consider A_1. We say that A_1 satisfies **PPAP** if and only if the condition (Def. 1) is satisfied.

(Def. 1) Let given M, N, a, b, c, a', b', c'. Suppose M is a line and N is a line and $a \in M$ and $b \in M$ and $c \in M$ and $a' \in N$ and $b' \in N$ and $c' \in N$ and $a,b' \upharpoonleft\upharpoonright b,a'$ and $b,c' \upharpoonleft\upharpoonright c,b'$. Then $a,c' \upharpoonleft\upharpoonright c,a'$.

We introduce A_1 satisfies **PPAP** as a synonym of A_1 satisfies **PPAP**.

Let A_1 be an affine space. We say that A_1 is Pappian if and only if the condition (Def. 2) is satisfied.

(Def. 2) Let M, N be subsets of A_1 and o, a, b, c, a', b', c' be elements of A_1. Suppose that M is a line and N is a line and $M \neq N$ and $o \in M$ and $o \in N$ and $o \neq a$ and $o \neq a'$ and $o \neq b$ and $o \neq b'$ and $o \neq c$ and $o \neq c'$ and $a \in M$ and $b \in M$ and $c \in M$ and $a' \in N$ and $b' \in N$ and $c' \in N$ and $a,b' \upharpoonleft\upharpoonright b,a'$ and $b,c' \upharpoonleft\upharpoonright c,b'$. Then $a,c' \upharpoonleft\upharpoonright c,a'$.

We introduce A_1 satisfies **PAP** as a synonym of A_1 is Pappian.

Let us consider A_1. We say that A_1 satisfies $\mathbf{PAP}_1$ if and only if the condition (Def. 3) is satisfied.

(Def. 3) Let given M, N, o, a, b, c, a', b', c'. Suppose that M is a line and N is a line and $M \neq N$ and $o \in M$ and $o \in N$ and $o \neq a$ and $o \neq a'$ and $o \neq b$ and $o \neq b'$ and $o \neq c$ and $o \neq c'$ and $a \in M$ and $b \in M$ and $c \in M$ and $b' \in N$ and $c' \in N$ and $a,b' \upharpoonleft\upharpoonright b,a'$ and $b,c' \upharpoonleft\upharpoonright c,b'$ and $a,c' \upharpoonleft\upharpoonright c,a'$ and $b \neq c$. Then $a' \in N$.

We introduce A_1 satisfies $\mathbf{PAP}_1$ as a synonym of A_1 satisfies $\mathbf{PAP}_1$.

Let A_1 be an affine space. We say that A_1 is Desarguesian if and only if the condition (Def. 4) is satisfied.

(Def. 4) Let A, P, C be subsets of A_1 and o, a, b, c, a', b', c' be elements of A_1. Suppose that $o \in A$ and $o \in P$ and $o \in C$ and $o \neq a$ and $o \neq b$ and $o \neq c$ and $a \in A$ and $a' \in A$ and $b \in P$ and $b' \in P$ and $c \in C$ and $c' \in C$ and A is a line and P is a line and C is a line and $A \neq P$ and $A \neq C$ and $a,b \upharpoonleft\upharpoonright a',b'$ and $a,c \upharpoonleft\upharpoonright a',c'$. Then $b,c \upharpoonleft\upharpoonright b',c'$.

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We introduce A_1 satisfies **DES** as a synonym of A_1 is Desarguesian.

Let us consider A_1. We say that A_1 satisfies $\mathbf{DES}_1$ if and only if the condition (Def. 5) is satisfied.

(Def. 5) Let given A, P, C, o, a, b, c, a', b', c'. Suppose that $o \in A$ and $o \in P$ and $o \neq a$ and $o \neq b$ and $o \neq c$ and $a \in A$ and $a' \in A$ and $b \in P$ and $b' \in P$ and $c \in C$ and $c' \in C$ and A is a line and P is a line and C is a line and $A \neq P$ and $A \neq C$ and $a,b \upuparrows a',b'$ and $a,c \upuparrows a',c'$ and $b,c \upuparrows b',c'$ and not $\mathbf{L}(a,b,c)$ and $c \neq c'$. Then $o \in C$.

We introduce A_1 satisfies $\mathbf{DES}_1$ as a synonym of A_1 satisfies $\mathbf{DES}_1$.

Let us consider A_1. We say that A_1 satisfies $\mathbf{DES}_2$ if and only if the condition (Def. 6) is satisfied.

(Def. 6) Let given A, P, C, o, a, b, c, a', b', c'. Suppose that $o \in A$ and $o \in P$ and $o \in C$ and $o \neq a$ and $o \neq b$ and $o \neq c$ and $a \in A$ and $a' \in A$ and $b \in P$ and $b' \in P$ and $c \in C$ and A is a line and P is a line and C is a line and $A \neq P$ and $A \neq C$ and $a,b \upuparrows a',b'$ and $a,c \upuparrows a',c'$ and $b,c \upuparrows b',c'$. Then $c' \in C$.

We introduce A_1 satisfies $\mathbf{DES}_2$ as a synonym of A_1 satisfies $\mathbf{DES}_2$.

Let A_1 be an affine space. We say that A_1 is Moufangian if and only if the condition (Def. 7) is satisfied.

(Def. 7) Let K be a subset of A_1 and o, a, b, c, a', b', c' be elements of A_1. Suppose K is a line and $o \in K$ and $c \in K$ and $c' \in K$ and $a \notin K$ and $o \neq c$ and $a \neq b$ and $\mathbf{L}(o,a,a')$ and $\mathbf{L}(o,b,b')$ and $a,b \upuparrows a',b'$ and $a,c \upuparrows a',c'$ and a, b // K. Then $b,c \upuparrows b',c'$.

We introduce A_1 satisfies **TDES** as a synonym of A_1 is Moufangian.

Let us consider A_1. We say that A_1 satisfies $\mathbf{TDES}_1$ if and only if the condition (Def. 8) is satisfied.

(Def. 8) Let given K, o, a, b, c, a', b', c'. Suppose K is a line and $o \in K$ and $c \in K$ and $c' \in K$ and $a \notin K$ and $o \neq c$ and $a \neq b$ and $\mathbf{L}(o,a,a')$ and $a,b \upuparrows a',b'$ and $b,c \upuparrows b',c'$ and $a,c \upuparrows a',c'$ and a, b // K. Then $\mathbf{L}(o,b,b')$.

We introduce A_1 satisfies $\mathbf{TDES}_1$ as a synonym of A_1 satisfies $\mathbf{TDES}_1$.

Let us consider A_1. We say that A_1 satisfies $\mathbf{TDES}_2$ if and only if the condition (Def. 9) is satisfied.

(Def. 9) Let given K, o, a, b, c, a', b', c'. Suppose K is a line and $o \in K$ and $c \in K$ and $c' \in K$ and $a \notin K$ and $o \neq c$ and $a \neq b$ and $\mathbf{L}(o,a,a')$ and $\mathbf{L}(o,b,b')$ and $b,c \upuparrows b',c'$ and $a,c \upuparrows a',c'$ and a, b // K. Then $a,b \upuparrows a',b'$.

We introduce A_1 satisfies $\mathbf{TDES}_2$ as a synonym of A_1 satisfies $\mathbf{TDES}_2$.

Let us consider A_1. We say that A_1 satisfies $\mathbf{TDES}_3$ if and only if the condition (Def. 10) is satisfied.

(Def. 10) Let given K, o, a, b, c, a', b', c'. Suppose K is a line and $o \in K$ and $c \in K$ and $a \notin K$ and $o \neq c$ and $a \neq b$ and $\mathbf{L}(o,a,a')$ and $\mathbf{L}(o,b,b')$ and $a,b \upuparrows a',b'$ and $a,c \upuparrows a',c'$ and $b,c \upuparrows b',c'$ and a, b // K. Then $c' \in K$.

We introduce A_1 satisfies $\mathbf{TDES}_3$ as a synonym of A_1 satisfies $\mathbf{TDES}_3$.

Let A_1 be an affine space. We say that A_1 is translational if and only if the condition (Def. 11) is satisfied.

(Def. 11) Let A, P, C be subsets of A_1 and a, b, c, a', b', c' be elements of A_1. Suppose that A // P and A // C and $a \in A$ and $a' \in A$ and $b \in P$ and $b' \in P$ and $c \in C$ and $c' \in C$ and A is a line and P is a line and C is a line and $A \neq P$ and $A \neq C$ and $a,b \upuparrows a',b'$ and $a,c \upuparrows a',c'$. Then $b,c \upuparrows b',c'$.

We introduce A_1 satisfies **des** as a synonym of A_1 is translational.

Let us consider A_1. We say that A_1 satisfies $\mathbf{des}_1$ if and only if the condition (Def. 12) is satisfied.

(Def. 12) Let given A, P, C, a, b, c, a', b', c'. Suppose that A // P and $a \in A$ and $a' \in A$ and $b \in P$ and $b' \in P$ and $c \in C$ and $c' \in C$ and A is a line and P is a line and C is a line and $A \neq P$ and $A \neq C$ and $a,b \upuparrows a',b'$ and $a,c \upuparrows a',c'$ and $b,c \upuparrows b',c'$ and not $\mathbf{L}(a,b,c)$ and $c \neq c'$. Then A // C.

We introduce A_1 satisfies $\mathbf{des}_1$ as a synonym of A_1 satisfies $\mathbf{des}_1$.

Let A_1 be an affine space. We say that A_1 satisfies pap if and only if the condition (Def. 13) is satisfied.

(Def. 13) Let M, N be subsets of A_1 and a, b, c, a', b', c' be elements of A_1. Suppose M is a line and N is a line and $a \in M$ and $b \in M$ and $c \in M$ and $M \parallel N$ and $M \neq N$ and $a' \in N$ and $b' \in N$ and $c' \in N$ and $a,b' \upuparrows b,a'$ and $b,c' \upuparrows c,b'$. Then $a,c' \upuparrows c,a'$.

We introduce A_1 satisfies **pap** as a synonym of A_1 satisfies pap.

Let us consider A_1. We say that A_1 satisfies $\mathbf{pap}_1$ if and only if the condition (Def. 14) is satisfied.

(Def. 14) Let given M, N, a, b, c, a', b', c'. Suppose that M is a line and N is a line and $a \in M$ and $b \in M$ and $c \in M$ and $M \parallel N$ and $M \neq N$ and $a' \in N$ and $b' \in N$ and $a,b' \upuparrows b,a'$ and $b,c' \upuparrows c,b'$ and $a,c' \upuparrows c,a'$ and $a' \neq b'$. Then $c' \in N$.

We introduce A_1 satisfies $\mathbf{pap}_1$ as a synonym of A_1 satisfies $\mathbf{pap}_1$.

The following propositions are true:

(15)[1] A_1 satisfies **PAP** iff A_1 satisfies $\mathbf{PAP}_1$.

(16) A_1 satisfies **DES** iff A_1 satisfies $\mathbf{DES}_1$.

(17) If A_1 satisfies **TDES**, then A_1 satisfies $\mathbf{TDES}_1$.

(18) If A_1 satisfies $\mathbf{TDES}_1$, then A_1 satisfies $\mathbf{TDES}_2$.

(19) If A_1 satisfies $\mathbf{TDES}_2$, then A_1 satisfies $\mathbf{TDES}_3$.

(20) If A_1 satisfies $\mathbf{TDES}_3$, then A_1 satisfies **TDES**.

(21) A_1 satisfies **des** iff A_1 satisfies $\mathbf{des}_1$.

(22) A_1 satisfies **pap** iff A_1 satisfies $\mathbf{pap}_1$.

(23) If A_1 satisfies **PAP**, then A_1 satisfies **pap**.

(24) A_1 satisfies **PPAP** iff A_1 satisfies **PAP** and A_1 satisfies **pap**.

(25) If A_1 satisfies **PAP**, then A_1 satisfies **DES**.

(26) If A_1 satisfies **DES**, then A_1 satisfies **TDES**.

(27) If A_1 satisfies $\mathbf{TDES}_1$, then A_1 satisfies $\mathbf{des}_1$.

(28) If A_1 satisfies **TDES**, then A_1 satisfies **des**.

(29) If A_1 satisfies **des**, then A_1 satisfies **pap**.

REFERENCES

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[2] Henryk Oryszczyszyn and Krzysztof Prażmowski. Ordered affine spaces defined in terms of directed parallelity — part I. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/diraf.html.

[1] The propositions (1)–(14) have been removed.

[3] Henryk Oryszczyszyn and Krzysztof Prażmowski. Parallelity and lines in affine spaces. *Journal of Formalized Mathematics*, 2, 1990. http://mizar.org/JFM/Vol2/aff_1.html.

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