JOURNAL OF FORMALIZED MATHEMATICS
Volume 1, Released 1989, Published 2003
Inst. of Computer Science, Univ. of Białystok

Some Properties of Functions Modul and Signum

Jan Popiołek
Warsaw University
Białystok

Summary. The article includes definitions and theorems concerning basic properties of the following functions: $|x|$ – modul of real number, sgn x – signum of real number.

MML Identifier: ABSVALUE.
WWW: http://mizar.org/JFM/Vol1/absvalue.html

The articles [1] and [2] provide the notation and terminology for this paper.

In this paper x, y, z, t are real numbers.

Let us consider x. The functor $|x|$ yielding a real number is defined by:

(Def. 1) $|x| = \begin{cases} x, & \text{if } 0 \leq x, \\ -x, & \text{otherwise.} \end{cases}$

Let us notice that the functor $|x|$ is projective.

Let x be a real number. Then $|x|$ is a real number.

We now state a number of propositions:

(5)[1] $0 \leq |x|$.

(6) If $x \neq 0$, then $0 < |x|$.

(7) $x = 0$ iff $|x| = 0$.

(9)[2] If $|x| = -x$ and $x \neq 0$, then $x < 0$.

(10) $|x \cdot y| = |x| \cdot |y|$.

(11) $-|x| \leq x$ and $x \leq |x|$.

(12) $-y \leq x$ and $x \leq y$ iff $|x| \leq y$.

(13) $|x+y| \leq |x| + |y|$.

(14) If $x \neq 0$, then $|x| \cdot |\frac{1}{x}| = 1$.

(15) $|\frac{1}{x}| = \frac{1}{|x|}$.

(16) $|\frac{x}{y}| = \frac{|x|}{|y|}$.

[1] The propositions (1)–(4) have been removed.
[2] The proposition (8) has been removed.

(17) $|x| = |-x|$.

(18) $|x| - |y| \leq |x - y|$.

(19) $|x - y| \leq |x| + |y|$.

(21)[3] If $|x| \leq z$ and $|y| \leq t$, then $|x + y| \leq z + t$.

(22) $||x| - |y|| \leq |x - y|$.

(24)[4] If $0 \leq x \cdot y$, then $|x + y| = |x| + |y|$.

(25) If $|x + y| = |x| + |y|$, then $0 \leq x \cdot y$.

(26) $\frac{|x+y|}{1+|x+y|} \leq \frac{|x|}{1+|x|} + \frac{|y|}{1+|y|}$.

Let us consider x. The functor $\operatorname{sgn} x$ is defined by:

(Def. 2) $$\operatorname{sgn} x = \begin{cases} 1, \text{ if } 0 < x, \\ -1, \text{ if } x < 0, \\ 0, \text{ otherwise.} \end{cases}$$

Let us consider x. One can check that $\operatorname{sgn} x$ is real.
Let x be a real number. Then $\operatorname{sgn} x$ is a real number.
We now state a number of propositions:

(31)[5] If $\operatorname{sgn} x = 1$, then $0 < x$.

(32) If $\operatorname{sgn} x = -1$, then $x < 0$.

(33) If $\operatorname{sgn} x = 0$, then $x = 0$.

(34) $x = |x| \cdot \operatorname{sgn} x$.

(35) $\operatorname{sgn}(x \cdot y) = \operatorname{sgn} x \cdot \operatorname{sgn} y$.

(36) $\operatorname{sgn} \operatorname{sgn} x = \operatorname{sgn} x$.

(37) $\operatorname{sgn}(x + y) \leq \operatorname{sgn} x + \operatorname{sgn} y + 1$.

(38) If $x \neq 0$, then $\operatorname{sgn} x \cdot \operatorname{sgn}(\frac{1}{x}) = 1$.

(39) $\frac{1}{\operatorname{sgn} x} = \operatorname{sgn}(\frac{1}{x})$.

(40) $(\operatorname{sgn} x + \operatorname{sgn} y) - 1 \leq \operatorname{sgn}(x + y)$.

(41) $\operatorname{sgn} x = \operatorname{sgn}(\frac{1}{x})$.

(42) $\operatorname{sgn}(\frac{x}{y}) = \frac{\operatorname{sgn} x}{\operatorname{sgn} y}$.

---

[3] The proposition (20) has been removed.
[4] The proposition (23) has been removed.
[5] The propositions (27)–(30) have been removed.

REFERENCES

[1] Grzegorz Bancerek. The ordinal numbers. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/ordinal1.html.

[2] Krzysztof Hryniewiecki. Basic properties of real numbers. *Journal of Formalized Mathematics*, 1, 1989. http://mizar.org/JFM/Vol1/real_1.html.

Received June 21, 1989

Published January 2, 2004