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On the Partial Product of Series and Related Basic Inequalities

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Summary. This article describes definition of partial product of series, introduced similarly to its related partial sum, as well as several important inequalities true for chosen special series.

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The notation and terminology used in this paper are introduced in the following articles: [1], [9], [10], [5], [2], [4], [6], [7], [8], and [3].

For simplicity, we adopt the following convention: a, b, c are positive real numbers, m, x, y, z are real numbers, n is a natural number, and s, s_1, s_2, s_3, s_4, s_5 are sequences of real numbers.

Let us consider x. Note that $|x|$ is non negative.

We now state a number of propositions:

(1) If $y > x$ and $x \geq 0$ and $m \geq 0$, then $\frac{x}{y} \leq \frac{x+m}{y+m}$.

(2) $\frac{a+b}{2} \geq \sqrt{a \cdot b}$.

(3) $\frac{b}{a} + \frac{a}{b} \geq 2$.

(4) $(\frac{x+y}{2})^{\mathbf{2}} \geq x \cdot y$.

(5) $\frac{x^{\mathbf{2}}+y^{\mathbf{2}}}{2} \geq (\frac{x+y}{2})^{\mathbf{2}}$.

(6) $x^{\mathbf{2}} + y^{\mathbf{2}} \geq 2 \cdot x \cdot y$.

(7) $\frac{x^{\mathbf{2}}+y^{\mathbf{2}}}{2} \geq x \cdot y$.

(8) $x^{\mathbf{2}} + y^{\mathbf{2}} \geq 2 \cdot |x| \cdot |y|$.

(9) $(x + y)^{\mathbf{2}} \geq 4 \cdot x \cdot y$.

(10) $x^{\mathbf{2}} + y^{\mathbf{2}} + z^{\mathbf{2}} \geq x \cdot y + y \cdot z + x \cdot z$.

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(11) $(x + y + z)^{\mathbf{2}} \geq 3 \cdot (x \cdot y + y \cdot z + x \cdot z)$.

(12) $a^3 + b^3 + c^3 \geq 3 \cdot a \cdot b \cdot c$.

(13) $\frac{a^3+b^3+c^3}{3} \geq a \cdot b \cdot c$.

(14) $(\frac{a}{b})^3 + (\frac{b}{c})^3 + (\frac{c}{a})^3 \geq \frac{b}{a} + \frac{c}{b} + \frac{a}{c}$.

(15) $a + b + c \geq 3 \cdot \sqrt[3]{a \cdot b \cdot c}$.

(16) $\frac{a+b+c}{3} \geq \sqrt[3]{a \cdot b \cdot c}$.

(17) If $x + y + z = 1$, then $x \cdot y + y \cdot z + x \cdot z \leq \frac{1}{3}$.

(18) If $x + y = 1$, then $x \cdot y \leq \frac{1}{4}$.

(19) If $x + y = 1$, then $x^{\mathbf{2}} + y^{\mathbf{2}} \geq \frac{1}{2}$.

(20) If $a + b = 1$, then $(1 + \frac{1}{a}) \cdot (1 + \frac{1}{b}) \geq 9$.

(21) If $x + y = 1$, then $x^3 + y^3 \geq \frac{1}{4}$.

(22) If $a + b = 1$, then $a^3 + b^3 < 1$.

(23) If $a + b = 1$, then $(a + \frac{1}{a}) \cdot (b + \frac{1}{b}) \geq \frac{25}{4}$.

(24) If $|x| \leq a$, then $x^{\mathbf{2}} \leq a^{\mathbf{2}}$.

(25) If $|x| \geq a$, then $x^{\mathbf{2}} \geq a^{\mathbf{2}}$.

(26) $||x| - |y|| \leq |x| + |y|$.

(27) If $a \cdot b \cdot c = 1$, then $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \geq \sqrt{a} + \sqrt{b} + \sqrt{c}$.

(28) If $x > 0$ and $y > 0$ and $z < 0$ and $x + y + z = 0$, then $(x^2 + y^2 + z^2)^3 \geq 6 \cdot (x^3 + y^3 + z^3)^{\mathbf{2}}$.

(29) If $a \geq 1$, then $a^b + a^c \geq 2 \cdot a^{\sqrt{b \cdot c}}$.

(30) If $a \geq b$ and $b \geq c$, then $a^a \cdot b^b \cdot c^c \geq (a \cdot b \cdot c)^{\frac{a+b+c}{3}}$.

(31) $(a + b)^{n+2} \geq a^{n+2} + (n + 2) \cdot a^{n+1} \cdot b$.

(32) $\frac{a^n+b^n}{2} \geq (\frac{a+b}{2})^n$.

(33) If for every n holds $s(n) > 0$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) > 0$.

(34) If for every n holds $s(n) \geq 0$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) \geq 0$.

(35) If for every n holds $s(n) < 0$, then $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) < 0$.

(36) If $s = s_1\, s_1$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) \geq 0$.

(37) If for every n holds $s(n) > 0$ and $s(n) > s(n-1)$, then $(n+1) \cdot s(n+1) > (\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n)$.

(38) If $s = s_1\, s_2$ and for every n holds $s_1(n) \geq 0$ and $s_2(n) \geq 0$, then for every n holds $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) \leq (\sum_{\alpha=0}^{\kappa}(s_1)(\alpha))_{\kappa \in \mathbb{N}}(n) \cdot (\sum_{\alpha=0}^{\kappa}(s_2)(\alpha))_{\kappa \in \mathbb{N}}(n)$.

(39) If $s = s_1\, s_2$ and for every n holds $s_1(n) < 0$ and $s_2(n) < 0$, then $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n) \leq (\sum_{\alpha=0}^{\kappa}(s_1)(\alpha))_{\kappa \in \mathbb{N}}(n) \cdot (\sum_{\alpha=0}^{\kappa}(s_2)(\alpha))_{\kappa \in \mathbb{N}}(n)$.

(40) For every n holds $|(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa \in \mathbb{N}}(n)| \leq (\sum_{\alpha=0}^{\kappa}|s|(\alpha))_{\kappa \in \mathbb{N}}(n)$.

(41) $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) \leq (\sum_{\alpha=0}^{\kappa}|s|(\alpha))_{\kappa\in\mathbb{N}}(n)$.

Let us consider s. The partial product of s yielding a sequence of real numbers is defined by the conditions (Def. 1).

(Def. 1)(i) (The partial product of $s)(0) = s(0)$, and

(ii) for every n holds (the partial product of $s)(n+1) =$ (the partial product of $s)(n) \cdot s(n+1)$.

We now state a number of propositions:

(42) If for every n holds $s(n) > 0$, then (the partial product of $s)(n) > 0$.

(43) If for every n holds $s(n) \geq 0$, then (the partial product of $s)(n) \geq 0$.

(44) Suppose that for every n holds $s(n) > 0$ and $s(n) < 1$. Let given n. Then (the partial product of $s)(n) > 0$ and (the partial product of $s)(n) < 1$.

(45) If for every n holds $s(n) \geq 1$, then for every n holds (the partial product of $s)(n) \geq 1$.

(46) Suppose that for every n holds $s_1(n) \geq 0$ and $s_2(n) \geq 0$. Let given n. Then (the partial product of $s_1)(n) +$ (the partial product of $s_2)(n) \leq$ (the partial product of $s_1 + s_2)(n)$.

(47) If for every n holds $s(n) = \frac{2\cdot n+1}{2\cdot n+2}$, then (the partial product of $s)(n) \leq \frac{1}{\sqrt{3\cdot n+4}}$.

(48) If for every n holds $s_1(n) = 1 + s(n)$ and $s(n) > -1$ and $s(n) < 0$, then for every n holds $1 + (\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) \leq$ (the partial product of $s_1)(n)$.

(49) If for every n holds $s_1(n) = 1 + s(n)$ and $s(n) \geq 0$, then for every n holds $1 + (\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) \leq$ (the partial product of $s_1)(n)$.

(50) If $s_3 = s_1\, s_2$ and $s_4 = s_1\, s_1$ and $s_5 = s_2\, s_2$, then for every n holds $(\sum_{\alpha=0}^{\kappa}(s_3)(\alpha))_{\kappa\in\mathbb{N}}(n)^{\mathbf{2}} \leq (\sum_{\alpha=0}^{\kappa}(s_4)(\alpha))_{\kappa\in\mathbb{N}}(n) \cdot (\sum_{\alpha=0}^{\kappa}(s_5)(\alpha))_{\kappa\in\mathbb{N}}(n)$.

(51) If $s_4 = s_1\, s_1$ and $s_5 = s_2\, s_2$ and for every n holds $s_1(n) \geq 0$ and $s_2(n) \geq 0$ and $s_3(n) = (s_1(n) + s_2(n))^{\mathbf{2}}$, then for every n holds $\sqrt{(\sum_{\alpha=0}^{\kappa}(s_3)(\alpha))_{\kappa\in\mathbb{N}}(n)} \leq \sqrt{(\sum_{\alpha=0}^{\kappa}(s_4)(\alpha))_{\kappa\in\mathbb{N}}(n)} + \sqrt{(\sum_{\alpha=0}^{\kappa}(s_5)(\alpha))_{\kappa\in\mathbb{N}}(n)}$.

(52) If for every n holds $s(n) > 0$ and $s(n) > s(n-1)$, then $(\sum_{\alpha=0}^{\kappa} s(\alpha))_{\kappa\in\mathbb{N}}(n) \geq (n+1) \cdot \sqrt[n+1]{(\text{the partial product of } s)(n)}$.

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