FORMALIZED MATHEMATICS
Volume 12, Number 2, 2004
University of Białystok

Concatenation of Finite Sequences Reducing Overlapping Part and an Argument of Separators of Sequential Files

Hirofumi Fukura
Shinshu University
Nagano

Yatsuka Nakamura
Shinshu University
Nagano

Summary. For two finite sequences, we present a notion of their concatenation, reducing overlapping part of the tail of the former and the head of the latter. At the same time, we also give a notion of common part of two finite sequences, which relates to the concatenation given here. A finite sequence is separated by another finite sequence (separator). We examined the condition that a separator separates uniquely any finite sequence. This will become a model of a separator of sequential files.

MML Identifier: FINSEQ_8.

The terminology and notation used here are introduced in the following articles: [14], [15], [9], [1], [12], [16], [3], [10], [2], [4], [5], [8], [13], [7], [11], and [6].

The following propositions are true:

(1) For every set D and for every finite sequence f of elements of D holds $f{\restriction}0 = \emptyset$.

(2) For every set D and for every finite sequence f of elements of D holds $f_{\downharpoonleft 0} = f$.

Let D be a set and let f, g be finite sequences of elements of D. Then $f \frown g$ is a finite sequence of elements of D.

Next we state three propositions:

(3) For every non empty set D and for all finite sequences f, g of elements of D such that $\mathrm{len}\, f \geqslant 1$ holds $\mathrm{mid}(f \frown g, 1, \mathrm{len}\, f) = f$.

(4) Let D be a set, f be a finite sequence of elements of D, and i be a natural number. If $i \geqslant \mathrm{len}\, f$, then $f_{\downharpoonleft i} = \varepsilon_D$.

ISSN 1426–2630

(5) For every non empty set D and for all natural numbers k_1, k_2 holds $\mathrm{mid}(\varepsilon_D, k_1, k_2) = \varepsilon_D$.

Let D be a set, let f be a finite sequence of elements of D, and let k_1, k_2 be natural numbers. The functor $\mathrm{smid}(f, k_1, k_2)$ yields a finite sequence of elements of D and is defined as follows:

(Def. 1) $\mathrm{smid}(f, k_1, k_2) = f_{\lfloor k_1 -' 1} \restriction ((k_2 + 1) -' k_1)$.

One can prove the following propositions:

(6) Let D be a non empty set, f be a finite sequence of elements of D, and k_1, k_2 be natural numbers. If $k_1 \leqslant k_2$, then $\mathrm{smid}(f, k_1, k_2) = \mathrm{mid}(f, k_1, k_2)$.

(7) Let D be a non empty set, f be a finite sequence of elements of D, and k_2 be a natural number. Then $\mathrm{smid}(f, 1, k_2) = f \restriction k_2$.

(8) Let D be a non empty set, f be a finite sequence of elements of D, and k_2 be a natural number. If $\mathrm{len}\, f \leqslant k_2$, then $\mathrm{smid}(f, 1, k_2) = f$.

(9) Let D be a set, f be a finite sequence of elements of D, and k_1, k_2 be natural numbers. If $k_1 > k_2$, then $\mathrm{smid}(f, k_1, k_2) = \emptyset$ and $\mathrm{smid}(f, k_1, k_2) = \varepsilon_D$.

(10) For every set D and for every finite sequence f of elements of D and for every natural number k_2 holds $\mathrm{smid}(f, 0, k_2) = \mathrm{smid}(f, 1, k_2 + 1)$.

(11) For every non empty set D and for all finite sequences f, g of elements of D holds $\mathrm{smid}(f \frown g, \mathrm{len}\, f + 1, \mathrm{len}\, f + \mathrm{len}\, g) = g$.

Let D be a non empty set and let f, g be finite sequences of elements of D. The functor $\mathrm{ovlpart}(f, g)$ yielding a finite sequence of elements of D is defined by the conditions (Def. 2).

(Def. 2)(i) $\mathrm{len}\, \mathrm{ovlpart}(f, g) \leqslant \mathrm{len}\, g$,

(ii) $\mathrm{ovlpart}(f, g) = \mathrm{smid}(g, 1, \mathrm{len}\, \mathrm{ovlpart}(f, g))$,

(iii) $\mathrm{ovlpart}(f, g) = \mathrm{smid}(f, (\mathrm{len}\, f -' \mathrm{len}\, \mathrm{ovlpart}(f, g)) + 1, \mathrm{len}\, f)$, and

(iv) for every natural number j such that $j \leqslant \mathrm{len}\, g$ and $\mathrm{smid}(g, 1, j) = \mathrm{smid}(f, (\mathrm{len}\, f -' j) + 1, \mathrm{len}\, f)$ holds $j \leqslant \mathrm{len}\, \mathrm{ovlpart}(f, g)$.

Next we state the proposition

(12) For every non empty set D and for all finite sequences f, g of elements of D holds $\mathrm{len}\, \mathrm{ovlpart}(f, g) \leqslant \mathrm{len}\, f$.

Let D be a non empty set and let f, g be finite sequences of elements of D. The functor $\mathrm{ovlcon}(f, g)$ yielding a finite sequence of elements of D is defined as follows:

(Def. 3) $\mathrm{ovlcon}(f, g) = f \frown (g_{\lfloor \mathrm{len}\, \mathrm{ovlpart}(f, g)})$.

One can prove the following proposition

(13) For every non empty set D and for all finite sequences f, g of elements of D holds $\mathrm{ovlcon}(f, g) = (f \restriction (\mathrm{len}\, f -' \mathrm{len}\, \mathrm{ovlpart}(f, g))) \frown g$.

Let D be a non empty set and let f, g be finite sequences of elements of D. The functor $\mathrm{ovlldiff}(f,g)$ yields a finite sequence of elements of D and is defined as follows:

(Def. 4) $\mathrm{ovlldiff}(f,g) = f{\restriction}(\mathrm{len}\, f -' \mathrm{len}\, \mathrm{ovlpart}(f,g))$.

Let D be a non empty set and let f, g be finite sequences of elements of D. The functor $\mathrm{ovlrdiff}(f,g)$ yields a finite sequence of elements of D and is defined by:

(Def. 5) $\mathrm{ovlrdiff}(f,g) = g_{\lfloor \mathrm{len}\, \mathrm{ovlpart}(f,g)}$.

One can prove the following propositions:

(14) Let D be a non empty set and f, g be finite sequences of elements of D. Then $\mathrm{ovlcon}(f,g) = (\mathrm{ovlldiff}(f,g)) \frown \mathrm{ovlpart}(f,g) \frown \mathrm{ovlrdiff}(f,g)$ and $\mathrm{ovlcon}(f,g) = (\mathrm{ovlldiff}(f,g)) \frown ((\mathrm{ovlpart}(f,g)) \frown \mathrm{ovlrdiff}(f,g))$.

(15) Let D be a non empty set and f be a finite sequence of elements of D. Then $\mathrm{ovlcon}(f,f) = f$ and $\mathrm{ovlpart}(f,f) = f$ and $\mathrm{ovlldiff}(f,f) = \emptyset$ and $\mathrm{ovlrdiff}(f,f) = \emptyset$.

(16) For every non empty set D and for all finite sequences f, g of elements of D holds $\mathrm{ovlpart}(f \frown g, g) = g$ and $\mathrm{ovlpart}(f, f \frown g) = f$.

(17) Let D be a non empty set and f, g be finite sequences of elements of D. Then $\mathrm{len}\, \mathrm{ovlcon}(f,g) = (\mathrm{len}\, f + \mathrm{len}\, g) - \mathrm{len}\, \mathrm{ovlpart}(f,g)$ and $\mathrm{len}\, \mathrm{ovlcon}(f,g) = (\mathrm{len}\, f + \mathrm{len}\, g) -' \mathrm{len}\, \mathrm{ovlpart}(f,g)$ and $\mathrm{len}\, \mathrm{ovlcon}(f,g) = \mathrm{len}\, f + (\mathrm{len}\, g -' \mathrm{len}\, \mathrm{ovlpart}(f,g))$.

(18) For every non empty set D and for all finite sequences f, g of elements of D holds $\mathrm{len}\, \mathrm{ovlpart}(f,g) \leqslant \mathrm{len}\, f$ and $\mathrm{len}\, \mathrm{ovlpart}(f,g) \leqslant \mathrm{len}\, g$.

Let D be a non empty set and let C_1 be a finite sequence of elements of D. We say that C_1 separates uniquely if and only if the condition (Def. 6) is satisfied.

(Def. 6) Let f be a finite sequence of elements of D and i, j be natural numbers. Suppose $1 \leqslant i$ and $i < j$ and $(j + \mathrm{len}\, C_1) -' 1 \leqslant \mathrm{len}\, f$ and $\mathrm{smid}(f, i, (i + \mathrm{len}\, C_1) -' 1) = \mathrm{smid}(f, j, (j + \mathrm{len}\, C_1) -' 1)$ and $\mathrm{smid}(f, i, (i + \mathrm{len}\, C_1) -' 1) = C_1$. Then $j -' i \geqslant \mathrm{len}\, C_1$.

The following proposition is true

(19) Let D be a non empty set and C_1 be a finite sequence of elements of D. Then C_1 separates uniquely if and only if $\mathrm{len}\, \mathrm{ovlpart}((C_1)_{\lfloor 1}, C_1) = 0$.

Let D be a non empty set, let f, g be finite sequences of elements of D, and let n be a natural number. We say that g is a substring of f if and only if:

(Def. 7) If $\mathrm{len}\, g > 0$, then there exists a natural number i such that $n \leqslant i$ and $i \leqslant \mathrm{len}\, f$ and $\mathrm{mid}(f, i, (i -' 1) + \mathrm{len}\, g) = g$.

We now state four propositions:

(20) Let D be a non empty set, f, g be finite sequences of elements of D, and n be a natural number. If $\operatorname{len} g = 0$, then g is a substring of f.

(21) Let D be a non empty set, f, g be finite sequences of elements of D, and n, m be natural numbers. If $m \geqslant n$ and g is a substring of f, then g is a substring of f.

(22) For every non empty set D and for every finite sequence f of elements of D such that $1 \leqslant \operatorname{len} f$ holds f is a substring of f.

(23) Let D be a non empty set and f, g be finite sequences of elements of D. If g is a substring of f, then g is a substring of f.

Let D be a non empty set and let f, g be finite sequences of elements of D. We say that g is a preposition of f if and only if:

(Def. 8) If $\operatorname{len} g > 0$, then $1 \leqslant \operatorname{len} f$ and $\operatorname{mid}(f, 1, \operatorname{len} g) = g$.

One can prove the following four propositions:

(24) Let D be a non empty set and f, g be finite sequences of elements of D. If $\operatorname{len} g = 0$, then g is a preposition of f.

(25) For every non empty set D holds every finite sequence f of elements of D is a preposition of f.

(26) Let D be a non empty set and f, g be finite sequences of elements of D. If g is a preposition of f, then $\operatorname{len} g \leqslant \operatorname{len} f$.

(27) Let D be a non empty set and f, g be finite sequences of elements of D. If $\operatorname{len} g > 0$ and g is a preposition of f, then $g(1) = f(1)$.

Let D be a non empty set and let f, g be finite sequences of elements of D. We say that g is a postposition of f if and only if:

(Def. 9) $\operatorname{Rev}(g)$ is a preposition of $\operatorname{Rev}(f)$.

Next we state several propositions:

(28) Let D be a non empty set and f, g be finite sequences of elements of D. If $\operatorname{len} g = 0$, then g is a postposition of f.

(29) Let D be a non empty set and f, g be finite sequences of elements of D. If g is a postposition of f, then $\operatorname{len} g \leqslant \operatorname{len} f$.

(30) Let D be a non empty set, f, g be finite sequences of elements of D, and n be a natural number. Suppose g is a postposition of f. If $\operatorname{len} g > 0$, then $\operatorname{len} g \leqslant \operatorname{len} f$ and $\operatorname{mid}(f, (\operatorname{len} f + 1) -' \operatorname{len} g, \operatorname{len} f) = g$.

(31) Let D be a non empty set, f, g be finite sequences of elements of D, and n be a natural number such that if $\operatorname{len} g > 0$, then $\operatorname{len} g \leqslant \operatorname{len} f$ and $\operatorname{mid}(f, (\operatorname{len} f + 1) -' \operatorname{len} g, \operatorname{len} f) = g$. Then g is a postposition of f.

(32) Let D be a non empty set, f, g be finite sequences of elements of D, and n be a natural number. If $\operatorname{len} g = 0$, then g is a preposition of f.

(33) Let D be a non empty set, f, g be finite sequences of elements of D, and n be a natural number. If $1 \leqslant \operatorname{len} f$ and g is a preposition of f, then g is

a substring of f.

(34) Let D be a non empty set, f, g be finite sequences of elements of D, and n be a natural number. Suppose g is not a substring of f. Let i be a natural number. If $n \leqslant i$ and $0 < i$, then $\mathrm{mid}(f, i, (i -' 1) + \mathrm{len}\, g) \neq g$.

Let D be a non empty set, let f, g be finite sequences of elements of D, and let n be a natural number. The functor $\mathrm{instr}(n, f)$ yielding a natural number is defined by the conditions (Def. 10).

(Def. 10)(i) If $\mathrm{instr}(n, f) \neq 0$, then $n \leqslant \mathrm{instr}(n, f)$ and g is a preposition of $f_{\restriction \mathrm{instr}(n,f) -' 1}$ and for every natural number j such that $j \geqslant n$ and $j > 0$ and g is a preposition of $f_{\restriction j -' 1}$ holds $j \geqslant \mathrm{instr}(n, f)$, and

(ii) if $\mathrm{instr}(n, f) = 0$, then g is not a substring of f.

Let D be a non empty set and let f, C_1 be finite sequences of elements of D. The functor $\mathrm{addcr}(f, C_1)$ yields a finite sequence of elements of D and is defined by:

(Def. 11) $\mathrm{addcr}(f, C_1) = \mathrm{ovlcon}(f, C_1)$.

Let D be a non empty set and let r, C_1 be finite sequences of elements of D. We say that r is terminated by C_1 if and only if:

(Def. 12) If $\mathrm{len}\, C_1 > 0$, then $\mathrm{len}\, r \geqslant \mathrm{len}\, C_1$ and $\mathrm{instr}(1, r) = (\mathrm{len}\, r + 1) -' \mathrm{len}\, C_1$.

The following proposition is true

(35) For every non empty set D holds every finite sequence f of elements of D is terminated by f.

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Received March 18, 2004