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High Speed Adder Algorithm with Radix-2^k Sub Signed-Digit Number

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Summary. In this article, a new adder algorithm using Radix-2^k sub signed-digit numbers is defined and properties for the hardware-realization is discussed.

Until now, we proposed Radix-2^k sub signed-digit numbers in consideration of the hardware realization. In this article, we proposed High Speed Adder Algorithm using this Radix-2^k sub signed-digit numbers. This method has two ways to speed up at hardware-realization. One is 'bit compare' at carry calculation, it is proposed in another article. Other is carry calculation between two numbers. We proposed that n digits Radix-2^k signed-digit numbers is expressed in $n+1$ digits Radix-2^k sub signed-digit numbers, and addition result of two $n+1$ digits Radix-2^k sub signed-digit numbers is expressed in $n+1$ digits. In this way, carry operation between two Radix-2^k sub signed-digit numbers can be processed at $n+1$ digit adder circuit and additional circuit to operate carry is not needed.

In the first section of this article, we prepared some useful theorems for operation of Radix-2^k numbers. In the second section, we proved some properties about carry on Radix-2^k sub signed-digit numbers. In the last section, we defined the new addition operation using Radix-2^k sub signed-digit numbers, and we clarified its correctness.

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The terminology and notation used here are introduced in the following articles: [11], [13], [12], [1], [4], [3], [10], [7], [2], [8], [5], [6], and [9].

1. Preliminaries

In this paper i, n, m, k, x, y are natural numbers.

The following proposition is true

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(1) For every natural number k such that $2 \leqslant k$ holds $2 < \mathrm{Radix}\,k$.

2. Carry Operation at $n+1$ Digits Radix-2^k Sub Signed-Digit Number

The following propositions are true:

(2) For all integers x, y and for every natural number k such that $3 \leqslant k$ holds $\mathrm{SDSubAddCarry}(\mathrm{SDSubAddCarry}(x,k) + \mathrm{SDSubAddCarry}(y,k),k) = 0$.

(3) If $2 \leqslant k$, then $\mathrm{DigA\_SDSub}(\mathrm{SD2SDSub}\,\mathrm{DecSD}(m,n,k),n+1) = \mathrm{SDSubAddCarry}(\mathrm{DigA}(\mathrm{DecSD}(m,n,k),n),k)$.

(4) If $2 \leqslant k$ and m is represented by 1, k, then $\mathrm{DigA\_SDSub}(\mathrm{SD2SDSub}\,\mathrm{DecSD}(m,1,k),1+1) = \mathrm{SDSubAddCarry}(m,k)$.

(5) Let k, x, n be natural numbers. Suppose $n \geqslant 1$ and $k \geqslant 3$ and x is represented by $n+1$, k. Then $\mathrm{DigA\_SDSub}(\mathrm{SD2SDSub}\,\mathrm{DecSD}(x \bmod (\mathrm{Radix}\,k)^n_{\mathbb{N}},n,k),n+1) = \mathrm{SDSubAddCarry}(\mathrm{DigA}(\mathrm{DecSD}(x,n,k),n),k)$.

(6) If $2 \leqslant k$ and m is represented by 1, k, then $\mathrm{DigA\_SDSub}(\mathrm{SD2SDSub}\,\mathrm{DecSD}(m,1,k),1) = m - \mathrm{SDSubAddCarry}(m,k) \cdot \mathrm{Radix}\,k$.

(7) Let k, x, n be natural numbers. Suppose $n \geqslant 1$ and $k \geqslant 2$ and x is represented by $n+1$, k. Then $((\mathrm{Radix}\,k)^n_{\mathbb{N}}) \cdot \mathrm{DigA\_SDSub}(\mathrm{SD2SDSub}\,\mathrm{DecSD}(x,n+1,k),n+1) = (((\mathrm{Radix}\,k)^n_{\mathbb{N}}) \cdot \mathrm{DigA}(\mathrm{DecSD}(x,n+1,k),n+1) - ((\mathrm{Radix}\,k)^{n+1}_{\mathbb{N}}) \cdot \mathrm{SDSubAddCarry}(\mathrm{DigA}(\mathrm{DecSD}(x,n+1,k),n+1),k)) + ((\mathrm{Radix}\,k)^n_{\mathbb{N}}) \cdot \mathrm{SDSubAddCarry}(\mathrm{DigA}(\mathrm{DecSD}(x,n+1,k),n),k)$.

3. Definition for Adder Operation on Radix-2^k Sub Signed-Digit Number

Let i, n, k be natural numbers, let x be a n-tuple of k −SD_Sub, and let y be a n-tuple of k −SD_Sub. Let us assume that $i \in \mathrm{Seg}\,n$ and $k \geqslant 2$. The functor $\mathrm{SDSubAddDigit}(x,y,i,k)$ yields an element of k −SD_Sub and is defined as follows:

(Def. 1) $\mathrm{SDSubAddDigit}(x,y,i,k) = \mathrm{SDSubAddData}(\mathrm{DigA\_SDSub}(x,i) + \mathrm{DigA\_SDSub}(y,i),k) + \mathrm{SDSubAddCarry}(\mathrm{DigA\_SDSub}(x,i -' 1) + \mathrm{DigA\_SDSub}(y,i -' 1),k)$.

Let n, k be natural numbers and let x, y be n-tuples of k −SD_Sub. The functor $x' +' y$ yields a n-tuple of k −SD_Sub and is defined by:

(Def. 2) For every i such that $i \in \mathrm{Seg}\,n$ holds $\mathrm{DigA\_SDSub}(x' +' y,i) = \mathrm{SDSubAddDigit}(x,y,i,k)$.

Next we state two propositions:

(8) For every i such that $i \in \operatorname{Seg} n$ holds if $2 \leqslant k$, then SDSubAddDigit(SD2SDSubDecSD($x, n+1, k$), SD2SDSubDecSD($y, n+1, k$), i, k) = SDSubAddDigit(SD2SDSubDecSD($x \bmod (\operatorname{Radix} k)^{n}_{\mathbb{N}}, n, k$), SD2SDSubDecSD($y \bmod (\operatorname{Radix} k)^{n}_{\mathbb{N}}, n, k$), i, k).

(9) Let given n. Suppose $n \geqslant 1$. Let given k, x, y. Suppose $k \geqslant 3$ and x is represented by n, k and y is represented by n, k. Then $x + y =$ SDSub2IntOut SD2SDSubDecSD(x, n, k)$' +'$ SD2SDSubDecSD(y, n, k).

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