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Classes of Independent Partitions

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Summary. The paper includes proofs of few theorems proved earlier by Shunichi Kobayashi in many different settings.

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The terminology and notation used in this paper have been introduced in the following articles: [1], [3], [4], [5], [9], [2], [10], [12], [11], [7], [6], and [8].

1. Preliminaries

Let X, Y be sets and let R, S be relations between X and Y. Let us observe that $R \subseteq S$ if and only if:

(Def. 1) For every element x of X and for every element y of Y such that $\langle x, y\rangle \in R$ holds $\langle x, y\rangle \in S$.

For simplicity, we adopt the following rules: Y is a non empty set, a is an element of $\mathit{Boolean}^Y$, G is a subset of PARTITIONS(Y), and P, Q are partitions of Y.

Let Y be a non empty set and let G be a non empty subset of PARTITIONS(Y). We see that the element of G is a partition of Y.

One can prove the following propositions:

(1) $\bigwedge \emptyset_{\mathrm{PARTITIONS}(Y)} = \mathcal{O}(Y)$.

(2) For all equivalence relations R, S of Y holds $R \cup S \subseteq R \cdot S$.

(3) For every binary relation R on Y holds $R \subseteq \nabla_Y$.

(4) For every equivalence relation R of Y holds $\nabla_Y \cdot R = \nabla_Y$ and $R \cdot \nabla_Y = \nabla_Y$.

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(5) For every partition P of Y and for all elements x, y of Y holds $\langle x, y\rangle \in \equiv_P$ iff $x \in \mathrm{EqClass}(y, P)$.

(6) Let P, Q, R be partitions of Y. Suppose $\equiv_R = \equiv_P \cdot \equiv_Q$. Let x, y be elements of Y. Then $x \in \mathrm{EqClass}(y, R)$ if and only if there exists an element z of Y such that $x \in \mathrm{EqClass}(z, P)$ and $z \in \mathrm{EqClass}(y, Q)$.

(7) Let R, S be binary relations and Y be a set. If R is reflexive in Y and S is reflexive in Y, then $R \cdot S$ is reflexive in Y.

(8) For every binary relation R and for every set Y such that R is reflexive in Y holds $Y \subseteq \mathrm{field}\, R$.

(9) For every set Y and for every binary relation R on Y such that R is reflexive in Y holds $Y = \mathrm{field}\, R$.

(10) For all equivalence relations R, S of Y such that $R \cdot S = S \cdot R$ holds $R \cdot S$ is an equivalence relation of Y.

2. Boolean-Valued Functions

The following propositions are true:

(11) For all elements a, b of $\mathit{Boolean}^Y$ such that $a \Subset b$ holds $\neg b \Subset \neg a$.

(12) For every element a of $\mathit{Boolean}^Y$ and for every subset G of PARTITIONS(Y) and for every partition A of Y holds $\forall_{a,A} G \Subset a$.

(13) Let a, b be elements of $\mathit{Boolean}^Y$, G be a subset of PARTITIONS(Y), and P be a partition of Y. If $a \Subset b$, then $\forall_{a,P} G \Subset \forall_{b,P} G$.

(14) For every element a of $\mathit{Boolean}^Y$ and for every subset G of PARTITIONS(Y) and for every partition A of Y holds $a \Subset \exists_{a,A} G$.

(15) Let a, b be elements of $\mathit{Boolean}^Y$, G be a subset of PARTITIONS(Y), and P be a partition of Y. If $a \Subset b$, then $\exists_{a,P} G \Subset \exists_{b,P} G$.

3. Independent Classes of Partitions

One can prove the following four propositions:

(16) If G is independent, then for all subsets P, Q of PARTITIONS(Y) such that $P \subseteq G$ and $Q \subseteq G$ holds $\equiv_{\bigwedge P} \cdot \equiv_{\bigwedge Q} = \equiv_{\bigwedge Q} \cdot \equiv_{\bigwedge P}$.

(17) If G is independent, then $\forall_{\forall_{a,P} G, Q} G = \forall_{\forall_{a,Q} G, P} G$.

(18) If G is independent, then $\exists_{\exists_{a,P} G, Q} G = \exists_{\exists_{a,Q} G, P} G$.

(19) Let a be an element of $\mathit{Boolean}^Y$, G be a subset of PARTITIONS(Y), and P, Q be partitions of Y. If G is independent, then $\exists_{\forall_{a,P} G, Q} G \Subset \forall_{\exists_{a,Q} G, P} G$.

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