FORMALIZED MATHEMATICS
Volume 9, Number 1, 2001
University of Białystok

Predicate Calculus for Boolean Valued Functions. Part XII

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Summary. In this paper, we proved some elementary predicate calculus formulae containing the quantifiers of Boolean valued functions with respect to partitions. Such a theory is an analogy of ordinary predicate logic.

MML Identifier: `BVFUNC24`.

The terminology and notation used here are introduced in the following articles: [11], [4], [6], [1], [8], [7], [2], [3], [5], [12], [10], and [9].

1. Preliminaries

For simplicity, we adopt the following convention: Y is a non empty set, a is an element of $\mathrm{BVF}(Y)$, G is a subset of $\mathrm{PARTITIONS}(Y)$, A, B, C, D, E, F, J, M, N are partitions of Y, and x, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9 are sets.

The following propositions are true:

(1) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $E \neq F$ and $E \neq J$ and $F \neq J$. Then $\mathrm{CompF}(A, G) = B \wedge C \wedge D \wedge E \wedge F \wedge J$.

(2) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $B \neq C$ and $B \neq D$

ISSN 1426–2630

and $B \neq E$ and $B \neq F$ and $B \neq J$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $E \neq F$ and $E \neq J$ and $F \neq J$. Then $\mathrm{CompF}(B, G) = A \wedge C \wedge D \wedge E \wedge F \wedge J$.

(3) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $E \neq F$ and $E \neq J$ and $F \neq J$. Then $\mathrm{CompF}(C, G) = A \wedge B \wedge D \wedge E \wedge F \wedge J$.

(4) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $E \neq F$ and $E \neq J$ and $F \neq J$. Then $\mathrm{CompF}(D, G) = A \wedge B \wedge C \wedge E \wedge F \wedge J$.

(5) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $E \neq F$ and $E \neq J$ and $F \neq J$. Then $\mathrm{CompF}(E, G) = A \wedge B \wedge C \wedge D \wedge F \wedge J$.

(6) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $E \neq F$ and $E \neq J$ and $F \neq J$. Then $\mathrm{CompF}(F, G) = A \wedge B \wedge C \wedge D \wedge E \wedge J$.

(7) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $E \neq F$ and $E \neq J$ and $F \neq J$. Then $\mathrm{CompF}(J, G) = A \wedge B \wedge C \wedge D \wedge E \wedge F$.

(8) Let A, B, C, D, E, F, J be sets, h be a function, and A', B', C', D', E', F', J' be sets. Suppose that
$A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $E \neq F$ and $E \neq J$ and $F \neq J$ and $h = (B \dot{\longmapsto} B') +\cdot (C \dot{\longmapsto} C') +\cdot (D \dot{\longmapsto} D') +\cdot (E \dot{\longmapsto} E') +\cdot (F \dot{\longmapsto} F') +\cdot (J \dot{\longmapsto} J') +\cdot (A \dot{\longmapsto} A')$. Then $h(A) = A'$ and $h(B) = B'$ and $h(C) = C'$ and $h(D) = D'$

and $h(E) = E'$ and $h(F) = F'$ and $h(J) = J'$.

(9) Let A, B, C, D, E, F, J be sets, h be a function, and A', B', C', D', E', F', J' be sets. Suppose that
$A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $E \neq F$ and $E \neq J$ and $F \neq J$ and $h = (B \longmapsto B') +\cdot (C \longmapsto C') +\cdot (D \longmapsto D') +\cdot (E \longmapsto E') +\cdot (F \longmapsto F') +\cdot (J \longmapsto J') +\cdot (A \longmapsto A')$. Then $\operatorname{dom} h = \{A, B, C, D, E, F, J\}$.

(10) Let A, B, C, D, E, F, J be sets, h be a function, and A', B', C', D', E', F', J' be sets. Suppose that
$A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $E \neq F$ and $E \neq J$ and $F \neq J$ and $h = (B \longmapsto B') +\cdot (C \longmapsto C') +\cdot (D \longmapsto D') +\cdot (E \longmapsto E') +\cdot (F \longmapsto F') +\cdot (J \longmapsto J') +\cdot (A \longmapsto A')$. Then $\operatorname{rng} h = \{h(A), h(B), h(C), h(D), h(E), h(F), h(J)\}$.

(11) Let G be a subset of PARTITIONS(Y), A, B, C, D, E, F, J be partitions of Y, z, u be elements of Y, and h be a function. Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $E \neq F$ and $E \neq J$ and $F \neq J$. Then $\mathrm{EqClass}(u, B \wedge C \wedge D \wedge E \wedge F \wedge J) \cap \mathrm{EqClass}(z, A) \neq \emptyset$.

(12) Let G be a subset of PARTITIONS(Y), A, B, C, D, E, F, J be partitions of Y, and z, u be elements of Y. Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $E \neq F$ and $E \neq J$ and $F \neq J$ and $\mathrm{EqClass}(z, C \wedge D \wedge E \wedge F \wedge J) = \mathrm{EqClass}(u, C \wedge D \wedge E \wedge F \wedge J)$. Then $\mathrm{EqClass}(u, \mathrm{CompF}(A, G)) \cap \mathrm{EqClass}(z, \mathrm{CompF}(B, G)) \neq \emptyset$.

(13) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$. Then $\mathrm{CompF}(A, G) = B \wedge C \wedge D \wedge E \wedge F \wedge J \wedge M$.

(14) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M\}$ and $A \neq B$ and $A \neq C$

and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$. Then $\mathrm{CompF}(B, G) = A \wedge C \wedge D \wedge E \wedge F \wedge J \wedge M$.

(15) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$. Then $\mathrm{CompF}(C, G) = A \wedge B \wedge D \wedge E \wedge F \wedge J \wedge M$.

(16) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$. Then $\mathrm{CompF}(D, G) = A \wedge B \wedge C \wedge E \wedge F \wedge J \wedge M$.

(17) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$. Then $\mathrm{CompF}(E, G) = A \wedge B \wedge C \wedge D \wedge F \wedge J \wedge M$.

(18) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$. Then $\mathrm{CompF}(F, G) = A \wedge B \wedge C \wedge D \wedge E \wedge J \wedge M$.

(19) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$. Then $\mathrm{CompF}(J, G) = A \wedge B \wedge C \wedge D \wedge E \wedge F \wedge M$.

(20) Suppose that

G is a coordinate and $G = \{A, B, C, D, E, F, J, M\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$. Then $\mathrm{CompF}(M, G) = A \wedge B \wedge C \wedge D \wedge E \wedge F \wedge J$.

(21) Let A, B, C, D, E, F, J, M be sets, h be a function, and A', B', C', D', E', F', J', M' be sets. Suppose that
$A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$ and $h = (B \mathrel{\dot{\longmapsto}} B') +\cdot (C \mathrel{\dot{\longmapsto}} C') +\cdot (D \mathrel{\dot{\longmapsto}} D') +\cdot (E \mathrel{\dot{\longmapsto}} E') +\cdot (F \mathrel{\dot{\longmapsto}} F') +\cdot (J \mathrel{\dot{\longmapsto}} J') +\cdot (M \mathrel{\dot{\longmapsto}} M') +\cdot (A \mathrel{\dot{\longmapsto}} A')$. Then $h(A) = A'$ and $h(B) = B'$ and $h(C) = C'$ and $h(D) = D'$ and $h(E) = E'$ and $h(F) = F'$ and $h(J) = J'$ and $h(M) = M'$.

(22) Let A, B, C, D, E, F, J, M be sets, h be a function, and A', B', C', D', E', F', J', M' be sets. Suppose that
$A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$ and $h = (B \mathrel{\dot{\longmapsto}} B') +\cdot (C \mathrel{\dot{\longmapsto}} C') +\cdot (D \mathrel{\dot{\longmapsto}} D') +\cdot (E \mathrel{\dot{\longmapsto}} E') +\cdot (F \mathrel{\dot{\longmapsto}} F') +\cdot (J \mathrel{\dot{\longmapsto}} J') +\cdot (M \mathrel{\dot{\longmapsto}} M') +\cdot (A \mathrel{\dot{\longmapsto}} A')$. Then $\operatorname{dom} h = \{A, B, C, D, E, F, J, M\}$.

(23) Let A, B, C, D, E, F, J, M be sets, h be a function, and A', B', C', D', E', F', J', M' be sets. Suppose that
$A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$ and $h = (B \mathrel{\dot{\longmapsto}} B') +\cdot (C \mathrel{\dot{\longmapsto}} C') +\cdot (D \mathrel{\dot{\longmapsto}} D') +\cdot (E \mathrel{\dot{\longmapsto}} E') +\cdot (F \mathrel{\dot{\longmapsto}} F') +\cdot (J \mathrel{\dot{\longmapsto}} J') +\cdot (M \mathrel{\dot{\longmapsto}} M') +\cdot (A \mathrel{\dot{\longmapsto}} A')$. Then $\operatorname{rng} h = \{h(A), h(B), h(C), h(D), h(E), h(F), h(J), h(M)\}$.

(24) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, A, B, C, D, E, F, J, M be partitions of Y, z, u be elements of Y, and h be a function. Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and

$B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$. Then $\mathrm{EqClass}(u, B \wedge C \wedge D \wedge E \wedge F \wedge J \wedge M) \cap \mathrm{EqClass}(z, A) \neq \emptyset$.

(25) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, A, B, C, D, E, F, J, M be partitions of Y, and z, u be elements of Y. Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$ and $\mathrm{EqClass}(z, C \wedge D \wedge E \wedge F \wedge J \wedge M) = \mathrm{EqClass}(u, C \wedge D \wedge E \wedge F \wedge J \wedge M)$. Then $\mathrm{EqClass}(u, \mathrm{CompF}(A, G)) \cap \mathrm{EqClass}(z, \mathrm{CompF}(B, G)) \neq \emptyset$.

The scheme *UI10* deals with a set $\mathcal{A}$, a set $\mathcal{B}$, a set $\mathcal{C}$, a set $\mathcal{D}$, a set $\mathcal{E}$, a set $\mathcal{F}$, a set $\mathcal{G}$, a set $\mathcal{H}$, a set $\mathcal{I}$, a set $\mathcal{J}$, and and states that:

$\mathcal{P}[\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{G}, \mathcal{H}, \mathcal{I}, \mathcal{J}]$

provided the following condition is satisfied:

- For all sets x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10} holds $\mathcal{P}[x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}]$.

Let us consider x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9.

The functor $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\}$ yielding a set is defined as follows:

(Def. 1) $x \in \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\}$ iff $x = x_1$ or $x = x_2$ or $x = x_3$ or $x = x_4$ or $x = x_5$ or $x = x_6$ or $x = x_7$ or $x = x_8$ or $x = x_9$.

We now state a number of propositions:

(26) $x \in \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\}$ iff $x = x_1$ or $x = x_2$ or $x = x_3$ or $x = x_4$ or $x = x_5$ or $x = x_6$ or $x = x_7$ or $x = x_8$ or $x = x_9$.

(27) $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\} = \{x_1\} \cup \{x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\}$.

(28) $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\} = \{x_1, x_2\} \cup \{x_3, x_4, x_5, x_6, x_7, x_8, x_9\}$.

(29) $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\} = \{x_1, x_2, x_3\} \cup \{x_4, x_5, x_6, x_7, x_8, x_9\}$.

(30) $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\} = \{x_1, x_2, x_3, x_4\} \cup \{x_5, x_6, x_7, x_8, x_9\}$.

(31) $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\} = \{x_1, x_2, x_3, x_4, x_5\} \cup \{x_6, x_7, x_8, x_9\}$.

(32) $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\} = \{x_1, x_2, x_3, x_4, x_5, x_6\} \cup \{x_7, x_8, x_9\}$.

(33) $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\} = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\} \cup \{x_8, x_9\}$.

(34) $\{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\} = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\} \cup \{x_9\}$.

(35) Let G be a subset of $\mathrm{PARTITIONS}(Y)$ and A, B, C, D, E, F, J, M, N be partitions of Y. Suppose that

G is a coordinate and $G = \{A, B, C, D, E, F, J, M, N\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$. Then $\mathrm{CompF}(A, G) = B \wedge C \wedge D \wedge E \wedge F \wedge J \wedge M \wedge N$.

(36) Let G be a subset of PARTITIONS(Y) and A, B, C, D, E, F, J, M, N be partitions of Y. Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M, N\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$. Then $\mathrm{CompF}(B, G) = A \wedge C \wedge D \wedge E \wedge F \wedge J \wedge M \wedge N$.

(37) Let G be a subset of PARTITIONS(Y) and A, B, C, D, E, F, J, M, N be partitions of Y. Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M, N\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$. Then $\mathrm{CompF}(C, G) = A \wedge B \wedge D \wedge E \wedge F \wedge J \wedge M \wedge N$.

(38) Let G be a subset of PARTITIONS(Y) and A, B, C, D, E, F, J, M, N be partitions of Y. Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M, N\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$. Then $\mathrm{CompF}(D, G) = A \wedge B \wedge C \wedge E \wedge F \wedge J \wedge M \wedge N$.

(39) Let G be a subset of PARTITIONS(Y) and A, B, C, D, E, F, J, M, N be partitions of Y. Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M, N\}$ and $A \neq B$ and

$A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$. Then $\mathrm{CompF}(E, G) = A \wedge B \wedge C \wedge D \wedge F \wedge J \wedge M \wedge N$.

(40) Let G be a subset of PARTITIONS(Y) and A, B, C, D, E, F, J, M, N be partitions of Y. Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M, N\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$. Then $\mathrm{CompF}(F, G) = A \wedge B \wedge C \wedge D \wedge E \wedge J \wedge M \wedge N$.

(41) Let G be a subset of PARTITIONS(Y) and A, B, C, D, E, F, J, M, N be partitions of Y. Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M, N\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$. Then $\mathrm{CompF}(J, G) = A \wedge B \wedge C \wedge D \wedge E \wedge F \wedge M \wedge N$.

(42) Let G be a subset of PARTITIONS(Y) and A, B, C, D, E, F, J, M, N be partitions of Y. Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M, N\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$. Then $\mathrm{CompF}(M, G) = A \wedge B \wedge C \wedge D \wedge E \wedge F \wedge J \wedge N$.

(43) Let G be a subset of PARTITIONS(Y) and A, B, C, D, E, F, J, M, N be partitions of Y. Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M, N\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and

$A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$. Then $\mathrm{CompF}(N, G) = A \wedge B \wedge C \wedge D \wedge E \wedge F \wedge J \wedge M$.

(44) Let A, B, C, D, E, F, J, M, N be sets, h be a function, and A', B', C', D', E', F', J', M', N' be sets. Suppose that
$A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$ and $h = (B \mapsto B') + \cdot (C \mapsto C') + \cdot (D \mapsto D') + \cdot (E \mapsto E') + \cdot (F \mapsto F') + \cdot (J \mapsto J') + \cdot (M \mapsto M') + \cdot (N \mapsto N') + \cdot (A \mapsto A')$. Then $h(A) = A'$ and $h(B) = B'$ and $h(C) = C'$ and $h(D) = D'$ and $h(E) = E'$ and $h(F) = F'$ and $h(J) = J'$ and $h(M) = M'$ and $h(N) = N'$.

(45) Let A, B, C, D, E, F, J, M, N be sets, h be a function, and A', B', C', D', E', F', J', M', N' be sets. Suppose that
$A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$ and $h = (B \mapsto B') + \cdot (C \mapsto C') + \cdot (D \mapsto D') + \cdot (E \mapsto E') + \cdot (F \mapsto F') + \cdot (J \mapsto J') + \cdot (M \mapsto M') + \cdot (N \mapsto N') + \cdot (A \mapsto A')$.
Then $\mathrm{dom}\, h = \{A, B, C, D, E, F, J, M, N\}$.

(46) Let A, B, C, D, E, F, J, M, N be sets, h be a function, and A', B', C', D', E', F', J', M', N' be sets. Suppose that
$A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$ and $h = (B \mapsto B') + \cdot (C \mapsto C') + \cdot (D \mapsto D') + \cdot (E \mapsto E') + \cdot (F \mapsto F') + \cdot (J \mapsto J') + \cdot (M \mapsto M') + \cdot (N \mapsto N') + \cdot (A \mapsto A')$.
Then $\mathrm{rng}\, h = \{h(A), h(B), h(C), h(D), h(E), h(F), h(J), h(M), h(N)\}$.

(47) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, A,

B, C, D, E, F, J, M, N be partitions of Y, z, u be elements of Y, and h be a function. Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M, N\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$. Then $\mathrm{EqClass}(u, B \wedge C \wedge D \wedge E \wedge F \wedge J \wedge M \wedge N) \cap \mathrm{EqClass}(z, A) \neq \emptyset$.

(48) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, A, B, C, D, E, F, J, M, N be partitions of Y, and z, u be elements of Y. Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M, N\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$ and $\mathrm{EqClass}(z, C \wedge D \wedge E \wedge F \wedge J \wedge M \wedge N) = \mathrm{EqClass}(u, C \wedge D \wedge E \wedge F \wedge J \wedge M \wedge N)$. Then $\mathrm{EqClass}(u, \mathrm{CompF}(A, G)) \cap \mathrm{EqClass}(z, \mathrm{CompF}(B, G)) \neq \emptyset$.

2. Predicate Calculus

We now state a number of propositions:

(49) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $E \neq F$ and $E \neq J$ and $F \neq J$. Then $\forall_{\forall_{a,A}G,B}G \Subset \forall_{\forall_{a,B}G,A}G$.

(50) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $E \neq F$ and $E \neq J$ and $F \neq J$. Then $\forall_{\forall_{a,A}G,B}G = \forall_{\forall_{a,B}G,A}G$.

(51) Suppose that

G is a coordinate and $G = \{A, B, C, D, E, F, J\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $E \neq F$ and $E \neq J$ and $F \neq J$. Then $\exists_{\forall_{a,A}G,B}G \Subset \forall_{\exists_{a,B}G,A}G$.

(52) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $E \neq F$ and $E \neq J$ and $F \neq J$. Then $\exists_{\exists_{a,B}G,A}G \Subset \exists_{\exists_{a,A}G,B}G$.

(53) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $E \neq F$ and $E \neq J$ and $F \neq J$. Then $\exists_{\exists_{a,A}G,B}G = \exists_{\exists_{a,B}G,A}G$.

(54) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $E \neq F$ and $E \neq J$ and $F \neq J$. Then $\forall_{\forall_{a,A}G,B}G \Subset \exists_{\forall_{a,B}G,A}G$.

(55) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $E \neq F$ and $E \neq J$ and $F \neq J$. Then $\forall_{\forall_{a,A}G,B}G \Subset \exists_{\exists_{a,B}G,A}G$.

(56) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $E \neq F$ and $E \neq J$ and $F \neq J$. Then $\forall_{\forall_{a,A}G,B}G \Subset \forall_{\exists_{a,B}G,A}G$.

(57) $\forall_{\exists_{a,A}G,B}G \Subset \exists_{\exists_{a,B}G,A}G$.

(58) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $E \neq F$ and $E \neq J$

and $F \neq J$. Then $\exists_{\forall_{a,A}G,B}G \Subset \exists_{\exists_{a,B}G,A}G$.

(59) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$. Then $\forall_{\forall_{a,A}G,B}G \Subset \forall_{\forall_{a,B}G,A}G$.

(60) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$. Then $\forall_{\forall_{a,A}G,B}G = \forall_{\forall_{a,B}G,A}G$.

(61) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$. Then $\exists_{\forall_{a,A}G,B}G \Subset \forall_{\exists_{a,B}G,A}G$.

(62) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$. Then $\exists_{\exists_{a,B}G,A}G \Subset \exists_{\exists_{a,A}G,B}G$.

(63) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$. Then $\exists_{\exists_{a,A}G,B}G = \exists_{\exists_{a,B}G,A}G$.

(64) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$

and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$. Then $\forall_{\forall_{a,A}G,B}G \Subset \exists_{\forall_{a,B}G,A}G$.

(66)[1] Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$. Then $\forall_{\forall_{a,A}G,B}G \Subset \forall_{\exists_{a,B}G,A}G$.

(67) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $F \neq J$ and $F \neq M$ and $J \neq M$. Then $\exists_{\forall_{a,A}G,B}G \Subset \exists_{\exists_{a,B}G,A}G$.

(68) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M, N\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$. Then $\forall_{\forall_{a,A}G,B}G \Subset \forall_{\forall_{a,B}G,A}G$.

(69) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M, N\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$. Then $\forall_{\forall_{a,A}G,B}G = \forall_{\forall_{a,B}G,A}G$.

(70) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M, N\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$

[1] The proposition (65) has been removed.

and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$. Then $\exists_{\forall_{a,A}G,B}G \Subset \forall_{\exists_{a,B}G,A}G$.

(71) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M, N\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$. Then $\exists_{\exists_{a,B}G,A}G \Subset \exists_{\exists_{a,A}G,B}G$.

(72) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M, N\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$. Then $\exists_{\exists_{a,A}G,B}G = \exists_{\exists_{a,B}G,A}G$.

(73) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M, N\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$. Then $\forall_{\forall_{a,A}G,B}G \Subset \exists_{\forall_{a,B}G,A}G$.

(74) $\forall_{\forall_{a,A}G,B}G \Subset \exists_{\exists_{a,B}G,A}G$.

(75) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F, J, M, N\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$. Then $\forall_{\forall_{a,A}G,B}G \Subset \forall_{\exists_{a,B}G,A}G$.

(76) Suppose that

G is a coordinate and $G = \{A, B, C, D, E, F, J, M, N\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $A \neq J$ and $A \neq M$ and $A \neq N$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $B \neq J$ and $B \neq M$ and $B \neq N$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $C \neq J$ and $C \neq M$ and $C \neq N$ and $D \neq E$ and $D \neq F$ and $D \neq J$ and $D \neq M$ and $D \neq N$ and $E \neq F$ and $E \neq J$ and $E \neq M$ and $E \neq N$ and $F \neq J$ and $F \neq M$ and $F \neq N$ and $J \neq M$ and $J \neq N$ and $M \neq N$. Then $\exists_{\forall_{a,A}G,B} G \Subset \exists_{\exists_{a,B}G,A} G$.

References

[1] Czesław Byliński. A classical first order language. *Formalized Mathematics*, 1(**4**):669–676, 1990.
[2] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.
[3] Czesław Byliński. The modification of a function by a function and the iteration of the composition of a function. *Formalized Mathematics*, 1(**3**):521–527, 1990.
[4] Shunichi Kobayashi and Kui Jia. A theory of Boolean valued functions and partitions. *Formalized Mathematics*, 7(**2**):249–254, 1998.
[5] Shunichi Kobayashi and Kui Jia. A theory of partitions. Part I. *Formalized Mathematics*, 7(**2**):243–247, 1998.
[6] Shunichi Kobayashi and Yatsuka Nakamura. A theory of Boolean valued functions and quantifiers with respect to partitions. *Formalized Mathematics*, 7(**2**):307–312, 1998.
[7] Konrad Raczkowski and Paweł Sadowski. Equivalence relations and classes of abstraction. *Formalized Mathematics*, 1(**3**):441–444, 1990.
[8] Andrzej Trybulec. Enumerated sets. *Formalized Mathematics*, 1(**1**):25–34, 1990.
[9] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.
[10] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(**1**):67–71, 1990.
[11] Zinaida Trybulec and Halina Święczkowska. Boolean properties of sets. *Formalized Mathematics*, 1(**1**):17–23, 1990.
[12] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1(**1**):73–83, 1990.

Received December 28, 1999