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Gauges

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The papers [20], [5], [23], [22], [10], [1], [17], [19], [24], [4], [2], [3], [21], [12], [11], [18], [7], [8], [9], [13], [14], [15], [6], and [16] provide the terminology and notation for this paper.

We follow the rules: i, i_1, i_2, j, j_1, j_2, k, m, n are natural numbers, D is a non empty set, and f is a finite sequence of elements of D.

We now state two propositions:

(1) If $\mathrm{len}\, f \geqslant 2$, then $f{\restriction}2 = \langle \pi_1 f, \pi_2 f \rangle$.

(2) If $k+1 \leqslant \mathrm{len}\, f$, then $f{\restriction}(k+1) = (f{\restriction}k) \frown \langle \pi_{k+1} f \rangle$.

In the sequel f denotes a finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$, G denotes a Go-board, and p denotes a point of $\mathcal{E}^2_{\mathrm{T}}$.

The following propositions are true:

(3) $\varepsilon_{(\text{the carrier of } \mathcal{E}^2_{\mathrm{T}})}$ is a sequence which elements belong to G.

(4) If f is a sequence which elements belong to G, then $f{\restriction}m$ is a sequence which elements belong to G.

(5) If f is a sequence which elements belong to G, then $f_{\lfloor m}$ is a sequence which elements belong to G.

(6) Suppose $1 \leqslant k$ and $k+1 \leqslant \mathrm{len}\, f$ and f is a sequence which elements belong to G. Then there exist natural numbers i_1, j_1, i_2, j_2 such that

(i) $\langle i_1, j_1 \rangle \in$ the indices of G,

(ii) $\pi_k f = G_{i_1, j_1}$,

(iii) $\langle i_2, j_2 \rangle \in$ the indices of G,

(iv) $\pi_{k+1} f = G_{i_2, j_2}$, and

(v) $i_1 = i_2$ and $j_1 + 1 = j_2$ or $i_1 + 1 = i_2$ and $j_1 = j_2$ or $i_1 = i_2 + 1$ and $j_1 = j_2$ or $i_1 = i_2$ and $j_1 = j_2 + 1$.

(7) Let f be a non empty finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$. Suppose f is a sequence which elements belong to G. Then f is standard and special.

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(8) Let f be a non empty finite sequence of elements of $\mathcal{E}^2_{\rm T}$. Suppose $\operatorname{len} f \geqslant 2$ and f is a sequence which elements belong to G. Then f is non constant.

(9) Let f be a non empty finite sequence of elements of $\mathcal{E}^2_{\rm T}$. Suppose that

(i) f is a sequence which elements belong to G,

(ii) there exist i, j such that $\langle i,\ j\rangle \in$ the indices of G and $p = G_{i,j}$, and

(iii) for all i_1, j_1, i_2, j_2 such that $\langle i_1,\ j_1\rangle \in$ the indices of G and $\langle i_2,\ j_2\rangle \in$ the indices of G and $\pi_{\operatorname{len} f} f = G_{i_1,j_1}$ and $p = G_{i_2,j_2}$ holds $|i_2-i_1|+|j_2-j_1| = 1$.

Then $f \frown \langle p\rangle$ is a sequence which elements belong to G.

(10) If $i + k < \operatorname{len} G$ and $1 \leqslant j$ and $j < \operatorname{width} G$ and $\operatorname{cell}(G, i, j)$ meets $\operatorname{cell}(G, i + k, j)$, then $k \leqslant 1$.

(11) For every non empty compact subset C of $\mathcal{E}^2_{\rm T}$ holds C is vertical iff E-bound $C \leqslant$ W-bound C.

(12) For every non empty compact subset C of $\mathcal{E}^2_{\rm T}$ holds C is horizontal iff N-bound $C \leqslant$ S-bound C.

Let C be a non empty subset of $\mathcal{E}^2_{\rm T}$ and let n be a natural number. The functor $\operatorname{Gauge}(C, n)$ yielding a matrix over $\mathcal{E}^2_{\rm T}$ is defined by the conditions (Def. 1).

(Def. 1)(i) $\operatorname{len} \operatorname{Gauge}(C, n) = 2^n + 3$,

(ii) $\operatorname{len} \operatorname{Gauge}(C, n) = \operatorname{width} \operatorname{Gauge}(C, n)$, and

(iii) for all i, j such that $\langle i,\ j\rangle \in$ the indices of $\operatorname{Gauge}(C, n)$ holds $(\operatorname{Gauge}(C, n))_{i,j} = [\text{W-bound}\, C + \frac{\text{E-bound}\, C - \text{W-bound}\, C}{2^n} \cdot (i-2), \text{S-bound}\, C + \frac{\text{N-bound}\, C - \text{S-bound}\, C}{2^n} \cdot (j-2)]$.

Let C be a compact non empty subset of $\mathcal{E}^2_{\rm T}$ and let n be a natural number. Note that $\operatorname{Gauge}(C, n)$ is non trivial line **X**-constant and column **Y**-constant.

In the sequel C is a compact non vertical non horizontal non empty subset of $\mathcal{E}^2_{\rm T}$.

Let us consider C, n. Observe that $\operatorname{Gauge}(C, n)$ is line **Y**-increasing and column **X**-increasing.

The following propositions are true:

(13) $\operatorname{len} \operatorname{Gauge}(C, n) \geqslant 4$.

(14) If $1 \leqslant j$ and $j \leqslant \operatorname{len} \operatorname{Gauge}(C, n)$, then $((\operatorname{Gauge}(C, n))_{2,j})_{\mathbf{1}} =$ W-bound C.

(15) If $1 \leqslant j$ and $j \leqslant \operatorname{len} \operatorname{Gauge}(C, n)$, then $((\operatorname{Gauge}(C, n))_{\operatorname{len} \operatorname{Gauge}(C,n) -' 1, j})_{\mathbf{1}} =$ E-bound C.

(16) If $1 \leqslant i$ and $i \leqslant \operatorname{len} \operatorname{Gauge}(C, n)$, then $((\operatorname{Gauge}(C, n))_{i,2})_{\mathbf{2}} =$ S-bound C.

(17) If $1 \leqslant i$ and $i \leqslant \operatorname{len} \operatorname{Gauge}(C, n)$, then $((\operatorname{Gauge}(C, n))_{i, \operatorname{len} \operatorname{Gauge}(C,n) -' 1})_{\mathbf{2}} =$ N-bound C.

(18) If $i \leqslant \operatorname{len} \operatorname{Gauge}(C, n)$, then $\operatorname{cell}(\operatorname{Gauge}(C, n), i, \operatorname{len} \operatorname{Gauge}(C, n)) \cap C = \emptyset$.

(19) If $j \leqslant \operatorname{len} \operatorname{Gauge}(C, n)$, then $\operatorname{cell}(\operatorname{Gauge}(C, n), \operatorname{len} \operatorname{Gauge}(C, n), j) \cap C = \emptyset$.

(20) If $i \leqslant \operatorname{len} \operatorname{Gauge}(C, n)$, then $\operatorname{cell}(\operatorname{Gauge}(C, n), i, 0) \cap C = \emptyset$.

(21) If $j \leqslant \operatorname{len} \operatorname{Gauge}(C, n)$, then $\operatorname{cell}(\operatorname{Gauge}(C, n), 0, j) \cap C = \emptyset$.

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