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Basic Properties of Genetic Algorithm

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Summary. We defined the set of the gene, the space treated by the genetic algorithm and the individual of the space. Moreover, we defined some genetic operators such as one point crossover and two points crossover, and the validity of many characters were proven.

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The terminology and notation used in this paper have been introduced in the following articles: [10], [6], [1], [4], [13], [12], [3], [8], [2], [11], [7], [9], and [5].

1. Definitions of Gene-Set, GA-Space and Individual

We follow the rules: D is a non empty set, f_1, f_2 are finite sequences of elements of D, and i, n, n_1, n_2, n_3, n_4, n_5, n_6 are natural numbers.

We now state two propositions:

(1) If $n \leqslant \operatorname{len} f_1$, then $(f_1 \frown f_2)_{\lfloor n} = ((f_1)_{\lfloor n}) \frown f_2$.

(2) $(f_1 \frown f_2) \restriction (\operatorname{len} f_1 + i) = f_1 \frown (f_2 \restriction i)$.

A Gene-Set is a non-empty non empty finite sequence.

Let S be a Gene-Set. We introduce $\mathrm{GA} - \mathrm{Space}\, S$ as a synonym of $\mathrm{Union}\, S$.

Let f be a non-empty non empty function. Note that $\mathrm{Union}\, f$ is non empty.

Let S be a Gene-Set. A finite sequence of elements of $\mathrm{GA} - \mathrm{Space}\, S$ is said to be a Individual of S if:

(Def. 1) $\operatorname{len} \mathrm{it} = \operatorname{len} S$ and for every i such that $i \in \operatorname{dom} \mathrm{it}$ holds $\mathrm{it}(i) \in S(i)$.

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2. Definitions of Several Genetic Operators

Let S be a Gene-Set, let p_1, p_2 be finite sequences of elements of $\mathrm{GA-Space}\,S$, and let us consider n. The functor $\mathrm{crossover}(p_1,p_2,n)$ yields a finite sequence of elements of $\mathrm{GA-Space}\,S$ and is defined as follows:

(Def. 2) $\mathrm{crossover}(p_1,p_2,n) = (p_1 \restriction n) \frown ((p_2)_{\lfloor n})$.

Let S be a Gene-Set, let p_1, p_2 be finite sequences of elements of $\mathrm{GA-Space}\,S$, and let us consider n_1, n_2. The functor $\mathrm{crossover}(p_1,p_2,n_1,n_2)$ yields a finite sequence of elements of $\mathrm{GA-Space}\,S$ and is defined as follows:

(Def. 3) $\mathrm{crossover}(p_1,p_2,n_1,n_2) =$
$\mathrm{crossover}(\mathrm{crossover}(p_1,p_2,n_1),\mathrm{crossover}(p_2,p_1,n_1),n_2)$.

Let S be a Gene-Set, let p_1, p_2 be finite sequences of elements of $\mathrm{GA-Space}\,S$, and let us consider n_1, n_2, n_3. The functor $\mathrm{crossover}(p_1,p_2,n_1,n_2,n_3)$ yields a finite sequence of elements of $\mathrm{GA-Space}\,S$ and is defined as follows:

(Def. 4) $\mathrm{crossover}(p_1,p_2,n_1,n_2,n_3) =$
$\mathrm{crossover}(\mathrm{crossover}(p_1,p_2,n_1,n_2),\mathrm{crossover}(p_2,p_1,n_1,n_2),n_3)$.

Let S be a Gene-Set, let p_1, p_2 be finite sequences of elements of $\mathrm{GA-Space}\,S$, and let us consider n_1, n_2, n_3, n_4. The functor $\mathrm{crossover}(p_1,p_2,n_1,n_2,n_3,n_4)$ yields a finite sequence of elements of $\mathrm{GA-Space}\,S$ and is defined as follows:

(Def. 5) $\mathrm{crossover}(p_1,p_2,n_1,n_2,n_3,n_4) =$
$\mathrm{crossover}(\mathrm{crossover}(p_1,p_2,n_1,n_2,n_3),\mathrm{crossover}(p_2,p_1,n_1,n_2,n_3),n_4)$.

Let S be a Gene-Set, let p_1, p_2 be finite sequences of elements of $\mathrm{GA-Space}\,S$, and let us consider n_1, n_2, n_3, n_4, n_5. The functor $\mathrm{crossover}(p_1,p_2,n_1,n_2,n_3,n_4,n_5)$ yielding a finite sequence of elements of $\mathrm{GA-Space}\,S$ is defined by:

(Def. 6) $\mathrm{crossover}(p_1,p_2,n_1,n_2,n_3,n_4,n_5) =$
$\mathrm{crossover}(\mathrm{crossover}(p_1,p_2,n_1,n_2,n_3,n_4),\mathrm{crossover}(p_2,p_1,n_1,n_2,n_3,n_4),n_5)$.

Let S be a Gene-Set, let p_1, p_2 be finite sequences of elements of $\mathrm{GA-Space}\,S$, and let us consider n_1, n_2, n_3, n_4, n_5, n_6. The functor $\mathrm{crossover}(p_1,p_2,n_1,n_2,n_3,n_4,n_5,n_6)$ yielding a finite sequence of elements of $\mathrm{GA-Space}\,S$ is defined as follows:

(Def. 7) $\mathrm{crossover}(p_1,p_2,n_1,n_2,n_3,n_4,n_5,n_6) =$
$\mathrm{crossover}(\mathrm{crossover}(p_1,p_2,n_1,n_2,n_3,n_4,n_5),$
$\mathrm{crossover}(p_2,p_1,n_1,n_2,n_3,n_4,n_5),n_6)$.

3. Properties of 1-point Crossover

In the sequel S denotes a Gene-Set and p_1, p_2 denote Individual of S.

The following proposition is true

(3) crossover(p_1, p_2, n) is a Individual of S.

Let S be a Gene-Set, let p_1, p_2 be Individual of S, and let us consider n. Then crossover(p_1, p_2, n) is a Individual of S.

One can prove the following propositions:

(4) crossover$(p_1, p_2, 0) = p_2$.

(5) If $n \geqslant \operatorname{len} p_1$, then crossover$(p_1, p_2, n) = p_1$.

4. Properties of 2-points Crossover

We now state the proposition

(6) crossover(p_1, p_2, n_1, n_2) is a Individual of S.

Let S be a Gene-Set, let p_1, p_2 be Individual of S, and let us consider n_1, n_2. Then crossover(p_1, p_2, n_1, n_2) is a Individual of S.

We now state several propositions:

(7) crossover$(p_1, p_2, 0, n)$ = crossover(p_2, p_1, n).

(8) crossover$(p_1, p_2, n, 0)$ = crossover(p_2, p_1, n).

(9) If $n_1 \geqslant \operatorname{len} p_1$, then crossover$(p_1, p_2, n_1, n_2)$ = crossover(p_1, p_2, n_2).

(10) If $n_2 \geqslant \operatorname{len} p_1$, then crossover$(p_1, p_2, n_1, n_2)$ = crossover(p_1, p_2, n_1).

(11) If $n_1 \geqslant \operatorname{len} p_1$ and $n_2 \geqslant \operatorname{len} p_1$, then crossover$(p_1, p_2, n_1, n_2) = p_1$.

(12) crossover$(p_1, p_2, n_1, n_1) = p_1$.

(13) crossover(p_1, p_2, n_1, n_2) = crossover(p_1, p_2, n_2, n_1).

5. Properties of 3-points Crossover

Next we state the proposition

(14) crossover$(p_1, p_2, n_1, n_2, n_3)$ is a Individual of S.

Let S be a Gene-Set, let p_1, p_2 be Individual of S, and let us consider n_1, n_2, n_3. Then crossover$(p_1, p_2, n_1, n_2, n_3)$ is a Individual of S.

We now state a number of propositions:

(15) crossover$(p_1, p_2, 0, n_2, n_3)$ = crossover(p_2, p_1, n_2, n_3) and crossover$(p_1, p_2, n_1, 0, n_3)$ = crossover(p_2, p_1, n_1, n_3) and crossover$(p_1, p_2, n_1, n_2, 0)$ = crossover(p_2, p_1, n_1, n_2).

(16) $\mathrm{crossover}(p_1, p_2, 0, 0, n_3) = \mathrm{crossover}(p_1, p_2, n_3)$ and $\mathrm{crossover}(p_1, p_2, n_1, 0, 0) = \mathrm{crossover}(p_1, p_2, n_1)$ and $\mathrm{crossover}(p_1, p_2, 0, n_2, 0) = \mathrm{crossover}(p_1, p_2, n_2)$.

(17) $\mathrm{crossover}(p_1, p_2, 0, 0, 0) = p_2$.

(18) If $n_1 \geqslant \mathrm{len}\, p_1$, then $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_3) = \mathrm{crossover}(p_1, p_2, n_2, n_3)$.

(19) If $n_2 \geqslant \mathrm{len}\, p_1$, then $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_3) = \mathrm{crossover}(p_1, p_2, n_1, n_3)$.

(20) If $n_3 \geqslant \mathrm{len}\, p_1$, then $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_3) = \mathrm{crossover}(p_1, p_2, n_1, n_2)$.

(21) If $n_1 \geqslant \mathrm{len}\, p_1$ and $n_2 \geqslant \mathrm{len}\, p_1$, then $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_3) = \mathrm{crossover}(p_1, p_2, n_3)$.

(22) If $n_1 \geqslant \mathrm{len}\, p_1$ and $n_3 \geqslant \mathrm{len}\, p_1$, then $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_3) = \mathrm{crossover}(p_1, p_2, n_2)$.

(23) If $n_2 \geqslant \mathrm{len}\, p_1$ and $n_3 \geqslant \mathrm{len}\, p_1$, then $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_3) = \mathrm{crossover}(p_1, p_2, n_1)$.

(24) If $n_1 \geqslant \mathrm{len}\, p_1$ and $n_2 \geqslant \mathrm{len}\, p_1$ and $n_3 \geqslant \mathrm{len}\, p_1$, then $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_3) = p_1$.

(25) $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_3) = \mathrm{crossover}(p_1, p_2, n_2, n_1, n_3)$ and $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_3) = \mathrm{crossover}(p_1, p_2, n_1, n_3, n_2)$.

(26) $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_3) = \mathrm{crossover}(p_1, p_2, n_3, n_1, n_2)$.

(27) $\mathrm{crossover}(p_1, p_2, n_1, n_1, n_3) = \mathrm{crossover}(p_1, p_2, n_3)$ and $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_1) = \mathrm{crossover}(p_1, p_2, n_2)$ and $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_2) = \mathrm{crossover}(p_1, p_2, n_1)$.

6. Properties of 4-points Crossover

Next we state the proposition

(28) $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_3, n_4)$ is a Individual of S.

Let S be a Gene-Set, let p_1, p_2 be Individual of S, and let us consider n_1, n_2, n_3, n_4. Then $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_3, n_4)$ is a Individual of S.

The following propositions are true:

(29) $\mathrm{crossover}(p_1, p_2, 0, n_2, n_3, n_4) = \mathrm{crossover}(p_2, p_1, n_2, n_3, n_4)$ and $\mathrm{crossover}(p_1, p_2, n_1, 0, n_3, n_4) = \mathrm{crossover}(p_2, p_1, n_1, n_3, n_4)$ and $\mathrm{crossover}(p_1, p_2, n_1, n_2, 0, n_4) = \mathrm{crossover}(p_2, p_1, n_1, n_2, n_4)$ and $\mathrm{crossover}(p_1, p_2, n_1, n_2, n_3, 0) = \mathrm{crossover}(p_2, p_1, n_1, n_2, n_3)$.

(30) $\mathrm{crossover}(p_1, p_2, 0, 0, n_3, n_4) = \mathrm{crossover}(p_1, p_2, n_3, n_4)$ and $\mathrm{crossover}(p_1, p_2, 0, n_2, 0, n_4) = \mathrm{crossover}(p_1, p_2, n_2, n_4)$ and $\mathrm{crossover}(p_1, p_2, 0, n_2, n_3, 0) = \mathrm{crossover}(p_1, p_2, n_2, n_3)$ and $\mathrm{crossover}(p_1, p_2, n_1, 0, n_3, 0) = \mathrm{crossover}(p_1, p_2, n_1, n_3)$ and $\mathrm{crossover}(p_1, p_2, n_1, 0, 0, n_4) = \mathrm{crossover}(p_1, p_2, n_1, n_4)$ and

$\text{crossover}(p_1, p_2, n_1, n_2, 0, 0) = \text{crossover}(p_1, p_2, n_1, n_2)$.

(31) $\text{crossover}(p_1, p_2, n_1, 0, 0, 0) = \text{crossover}(p_2, p_1, n_1)$ and $\text{crossover}(p_1, p_2, 0, n_2, 0, 0) = \text{crossover}(p_2, p_1, n_2)$ and $\text{crossover}(p_1, p_2, 0, 0, n_3, 0) = \text{crossover}(p_2, p_1, n_3)$ and $\text{crossover}(p_1, p_2, 0, 0, 0, n_4) = \text{crossover}(p_2, p_1, n_4)$.

(32) $\text{crossover}(p_1, p_2, 0, 0, 0, 0) = p_1$.

(33)(i) If $n_1 \geqslant \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4) = \text{crossover}(p_1, p_2, n_2, n_3, n_4)$,

(ii) if $n_2 \geqslant \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4) = \text{crossover}(p_1, p_2, n_1, n_3, n_4)$,

(iii) if $n_3 \geqslant \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4) = \text{crossover}(p_1, p_2, n_1, n_2, n_4)$, and

(iv) if $n_4 \geqslant \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4) = \text{crossover}(p_1, p_2, n_1, n_2, n_3)$.

(34)(i) If $n_1 \geqslant \text{len}\, p_1$ and $n_2 \geqslant \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4) = \text{crossover}(p_1, p_2, n_3, n_4)$,

(ii) if $n_1 \geqslant \text{len}\, p_1$ and $n_3 \geqslant \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4) = \text{crossover}(p_1, p_2, n_2, n_4)$,

(iii) if $n_1 \geqslant \text{len}\, p_1$ and $n_4 \geqslant \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4) = \text{crossover}(p_1, p_2, n_2, n_3)$,

(iv) if $n_2 \geqslant \text{len}\, p_1$ and $n_3 \geqslant \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4) = \text{crossover}(p_1, p_2, n_1, n_4)$,

(v) if $n_2 \geqslant \text{len}\, p_1$ and $n_4 \geqslant \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4) = \text{crossover}(p_1, p_2, n_1, n_3)$, and

(vi) if $n_3 \geqslant \text{len}\, p_1$ and $n_4 \geqslant \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4) = \text{crossover}(p_1, p_2, n_1, n_2)$.

(35)(i) If $n_1 \geqslant \text{len}\, p_1$ and $n_2 \geqslant \text{len}\, p_1$ and $n_3 \geqslant \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4) = \text{crossover}(p_1, p_2, n_4)$,

(ii) if $n_1 \geqslant \text{len}\, p_1$ and $n_2 \geqslant \text{len}\, p_1$ and $n_4 \geqslant \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4) = \text{crossover}(p_1, p_2, n_3)$,

(iii) if $n_1 \geqslant \text{len}\, p_1$ and $n_3 \geqslant \text{len}\, p_1$ and $n_4 \geqslant \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4) = \text{crossover}(p_1, p_2, n_2)$, and

(iv) if $n_2 \geqslant \text{len}\, p_1$ and $n_3 \geqslant \text{len}\, p_1$ and $n_4 \geqslant \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4) = \text{crossover}(p_1, p_2, n_1)$.

(36) If $n_1 \geqslant \text{len}\, p_1$ and $n_2 \geqslant \text{len}\, p_1$ and $n_3 \geqslant \text{len}\, p_1$ and $n_4 \geqslant \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4) = p_1$.

(37) $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4) = \text{crossover}(p_1, p_2, n_1, n_2, n_4, n_3)$ and $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4) = \text{crossover}(p_1, p_2, n_1, n_3, n_2, n_4)$ and $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4) = \text{crossover}(p_1, p_2, n_1, n_3, n_4, n_2)$ and $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4) = \text{crossover}(p_1, p_2, n_1, n_4, n_2, n_3)$ and $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4) = \text{crossover}(p_1, p_2, n_1, n_4, n_3, n_2)$ and

crossover$(p_1, p_2, n_1, n_2, n_3, n_4)$ = crossover$(p_1, p_2, n_2, n_1, n_3, n_4)$ and
crossover$(p_1, p_2, n_1, n_2, n_3, n_4)$ = crossover$(p_1, p_2, n_2, n_1, n_4, n_3)$ and
crossover$(p_1, p_2, n_1, n_2, n_3, n_4)$ = crossover$(p_1, p_2, n_2, n_3, n_1, n_4)$ and
crossover$(p_1, p_2, n_1, n_2, n_3, n_4)$ = crossover$(p_1, p_2, n_2, n_3, n_4, n_1)$ and
crossover$(p_1, p_2, n_1, n_2, n_3, n_4)$ = crossover$(p_1, p_2, n_2, n_4, n_1, n_3)$ and
crossover$(p_1, p_2, n_1, n_2, n_3, n_4)$ = crossover$(p_1, p_2, n_2, n_4, n_3, n_1)$ and
crossover$(p_1, p_2, n_1, n_2, n_3, n_4)$ = crossover$(p_1, p_2, n_3, n_1, n_2, n_4)$ and
crossover$(p_1, p_2, n_1, n_2, n_3, n_4)$ = crossover$(p_1, p_2, n_3, n_1, n_4, n_2)$ and
crossover$(p_1, p_2, n_1, n_2, n_3, n_4)$ = crossover$(p_1, p_2, n_3, n_2, n_1, n_4)$ and
crossover$(p_1, p_2, n_1, n_2, n_3, n_4)$ = crossover$(p_1, p_2, n_3, n_2, n_4, n_1)$ and
crossover$(p_1, p_2, n_1, n_2, n_3, n_4)$ = crossover$(p_1, p_2, n_3, n_4, n_1, n_2)$ and
crossover$(p_1, p_2, n_1, n_2, n_3, n_4)$ = crossover$(p_1, p_2, n_3, n_4, n_2, n_1)$ and
crossover$(p_1, p_2, n_1, n_2, n_3, n_4)$ = crossover$(p_1, p_2, n_4, n_1, n_2, n_3)$ and
crossover$(p_1, p_2, n_1, n_2, n_3, n_4)$ = crossover$(p_1, p_2, n_4, n_1, n_3, n_2)$ and
crossover$(p_1, p_2, n_1, n_2, n_3, n_4)$ = crossover$(p_1, p_2, n_4, n_2, n_1, n_3)$ and
crossover$(p_1, p_2, n_1, n_2, n_3, n_4)$ = crossover$(p_1, p_2, n_4, n_2, n_3, n_1)$ and
crossover$(p_1, p_2, n_1, n_2, n_3, n_4)$ = crossover$(p_1, p_2, n_4, n_3, n_1, n_2)$ and
crossover$(p_1, p_2, n_1, n_2, n_3, n_4)$ = crossover$(p_1, p_2, n_4, n_3, n_2, n_1)$.

(38) crossover$(p_1, p_2, n_1, n_1, n_3, n_4)$ = crossover(p_1, p_2, n_3, n_4) and
crossover$(p_1, p_2, n_1, n_2, n_1, n_4)$ = crossover(p_1, p_2, n_2, n_4) and
crossover$(p_1, p_2, n_1, n_2, n_3, n_1)$ = crossover(p_1, p_2, n_2, n_3) and
crossover$(p_1, p_2, n_1, n_2, n_2, n_4)$ = crossover(p_1, p_2, n_1, n_4) and
crossover$(p_1, p_2, n_1, n_2, n_3, n_2)$ = crossover(p_1, p_2, n_1, n_3) and
crossover$(p_1, p_2, n_1, n_2, n_3, n_3)$ = crossover(p_1, p_2, n_1, n_2).

(39) crossover$(p_1, p_2, n_1, n_1, n_3, n_3) = p_1$ and crossover$(p_1, p_2, n_1, n_2, n_1, n_2) = p_1$ and crossover$(p_1, p_2, n_1, n_2, n_2, n_1) = p_1$.

7. Properties of 5-points Crossover

Next we state the proposition

(40) crossover$(p_1, p_2, n_1, n_2, n_3, n_4, n_5)$ is a Individual of S.

Let S be a Gene-Set, let p_1, p_2 be Individual of S, and let us consider n_1, n_2, n_3, n_4, n_5. Then crossover$(p_1, p_2, n_1, n_2, n_3, n_4, n_5)$ is a Individual of S.

Next we state a number of propositions:

(41) crossover$(p_1, p_2, 0, n_2, n_3, n_4, n_5)$ = crossover$(p_2, p_1, n_2, n_3, n_4, n_5)$ and
crossover$(p_1, p_2, n_1, 0, n_3, n_4, n_5)$ = crossover$(p_2, p_1, n_1, n_3, n_4, n_5)$ and
crossover$(p_1, p_2, n_1, n_2, 0, n_4, n_5)$ = crossover$(p_2, p_1, n_1, n_2, n_4, n_5)$ and
crossover$(p_1, p_2, n_1, n_2, n_3, 0, n_5)$ = crossover$(p_2, p_1, n_1, n_2, n_3, n_5)$ and
crossover$(p_1, p_2, n_1, n_2, n_3, n_4, 0)$ = crossover$(p_2, p_1, n_1, n_2, n_3, n_4)$.

(42) $\text{crossover}(p_1, p_2, 0, 0, n_3, n_4, n_5) = \text{crossover}(p_1, p_2, n_3, n_4, n_5)$ and
$\text{crossover}(p_1, p_2, 0, n_2, 0, n_4, n_5) = \text{crossover}(p_1, p_2, n_2, n_4, n_5)$ and
$\text{crossover}(p_1, p_2, 0, n_2, n_3, 0, n_5) = \text{crossover}(p_1, p_2, n_2, n_3, n_5)$ and
$\text{crossover}(p_1, p_2, 0, n_2, n_3, n_4, 0) = \text{crossover}(p_1, p_2, n_2, n_3, n_4)$ and
$\text{crossover}(p_1, p_2, n_1, 0, 0, n_4, n_5) = \text{crossover}(p_1, p_2, n_1, n_4, n_5)$ and
$\text{crossover}(p_1, p_2, n_1, 0, n_3, 0, n_5) = \text{crossover}(p_1, p_2, n_1, n_3, n_5)$ and
$\text{crossover}(p_1, p_2, n_1, 0, n_3, n_4, 0) = \text{crossover}(p_1, p_2, n_1, n_3, n_4)$ and
$\text{crossover}(p_1, p_2, n_1, n_2, 0, 0, n_5) = \text{crossover}(p_1, p_2, n_1, n_2, n_5)$ and
$\text{crossover}(p_1, p_2, n_1, n_2, 0, n_4, 0) = \text{crossover}(p_1, p_2, n_1, n_2, n_4)$ and
$\text{crossover}(p_1, p_2, n_1, n_2, n_3, 0, 0) = \text{crossover}(p_1, p_2, n_1, n_2, n_3)$.

(43) $\text{crossover}(p_1, p_2, 0, 0, 0, n_4, n_5) = \text{crossover}(p_2, p_1, n_4, n_5)$ and
$\text{crossover}(p_1, p_2, 0, 0, n_3, 0, n_5) = \text{crossover}(p_2, p_1, n_3, n_5)$ and
$\text{crossover}(p_1, p_2, 0, 0, n_3, n_4, 0) = \text{crossover}(p_2, p_1, n_3, n_4)$ and
$\text{crossover}(p_1, p_2, 0, n_2, 0, 0, n_5) = \text{crossover}(p_2, p_1, n_2, n_5)$ and
$\text{crossover}(p_1, p_2, 0, n_2, 0, n_4, 0) = \text{crossover}(p_2, p_1, n_2, n_4)$ and
$\text{crossover}(p_1, p_2, 0, n_2, n_3, 0, 0) = \text{crossover}(p_2, p_1, n_2, n_3)$ and
$\text{crossover}(p_1, p_2, n_1, 0, 0, 0, n_5) = \text{crossover}(p_2, p_1, n_1, n_5)$ and
$\text{crossover}(p_1, p_2, n_1, 0, 0, n_4, 0) = \text{crossover}(p_2, p_1, n_1, n_4)$ and
$\text{crossover}(p_1, p_2, n_1, 0, n_3, 0, 0) = \text{crossover}(p_2, p_1, n_1, n_3)$ and
$\text{crossover}(p_1, p_2, n_1, n_2, 0, 0, 0) = \text{crossover}(p_2, p_1, n_1, n_2)$.

(44) $\text{crossover}(p_1, p_2, 0, 0, 0, 0, n_5) = \text{crossover}(p_1, p_2, n_5)$ and
$\text{crossover}(p_1, p_2, 0, 0, 0, n_4, 0) = \text{crossover}(p_1, p_2, n_4)$ and
$\text{crossover}(p_1, p_2, 0, 0, n_3, 0, 0) = \text{crossover}(p_1, p_2, n_3)$ and
$\text{crossover}(p_1, p_2, 0, n_2, 0, 0, 0) = \text{crossover}(p_1, p_2, n_2)$ and
$\text{crossover}(p_1, p_2, n_1, 0, 0, 0, 0) = \text{crossover}(p_1, p_2, n_1)$.

(45) $\text{crossover}(p_1, p_2, 0, 0, 0, 0, 0) = p_2$.

(46)(i) If $n_1 \geqslant \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \text{crossover}(p_1, p_2, n_2, n_3, n_4, n_5)$,

(ii) if $n_2 \geqslant \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \text{crossover}(p_1, p_2, n_1, n_3, n_4, n_5)$,

(iii) if $n_3 \geqslant \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \text{crossover}(p_1, p_2, n_1, n_2, n_4, n_5)$,

(iv) if $n_4 \geqslant \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \text{crossover}(p_1, p_2, n_1, n_2, n_3, n_5)$, and

(v) if $n_5 \geqslant \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4)$.

(47)(i) If $n_1 \geqslant \text{len}\, p_1$ and $n_2 \geqslant \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \text{crossover}(p_1, p_2, n_3, n_4, n_5)$,

(ii) if $n_1 \geqslant \text{len}\, p_1$ and $n_3 \geqslant \text{len}\, p_1$, then $\text{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \text{crossover}(p_1, p_2, n_2, n_4, n_5)$,

(iii) if $n_1 \geqslant \operatorname{len} p_1$ and $n_4 \geqslant \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_2, n_3, n_5)$,

(iv) if $n_1 \geqslant \operatorname{len} p_1$ and $n_5 \geqslant \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_2, n_3, n_4)$,

(v) if $n_2 \geqslant \operatorname{len} p_1$ and $n_3 \geqslant \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_1, n_4, n_5)$,

(vi) if $n_2 \geqslant \operatorname{len} p_1$ and $n_4 \geqslant \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_1, n_3, n_5)$,

(vii) if $n_2 \geqslant \operatorname{len} p_1$ and $n_5 \geqslant \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_1, n_3, n_4)$,

(viii) if $n_3 \geqslant \operatorname{len} p_1$ and $n_4 \geqslant \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_1, n_2, n_5)$,

(ix) if $n_3 \geqslant \operatorname{len} p_1$ and $n_5 \geqslant \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_1, n_2, n_4)$, and

(x) if $n_4 \geqslant \operatorname{len} p_1$ and $n_5 \geqslant \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_1, n_2, n_3)$.

(48)(i) If $n_1 \geqslant \operatorname{len} p_1$ and $n_2 \geqslant \operatorname{len} p_1$ and $n_3 \geqslant \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_4, n_5)$,

(ii) if $n_1 \geqslant \operatorname{len} p_1$ and $n_2 \geqslant \operatorname{len} p_1$ and $n_4 \geqslant \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_3, n_5)$,

(iii) if $n_1 \geqslant \operatorname{len} p_1$ and $n_2 \geqslant \operatorname{len} p_1$ and $n_5 \geqslant \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_3, n_4)$,

(iv) if $n_1 \geqslant \operatorname{len} p_1$ and $n_3 \geqslant \operatorname{len} p_1$ and $n_4 \geqslant \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_2, n_5)$,

(v) if $n_1 \geqslant \operatorname{len} p_1$ and $n_3 \geqslant \operatorname{len} p_1$ and $n_5 \geqslant \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_2, n_4)$,

(vi) if $n_1 \geqslant \operatorname{len} p_1$ and $n_4 \geqslant \operatorname{len} p_1$ and $n_5 \geqslant \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_2, n_3)$,

(vii) if $n_2 \geqslant \operatorname{len} p_1$ and $n_3 \geqslant \operatorname{len} p_1$ and $n_4 \geqslant \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_1, n_5)$,

(viii) if $n_2 \geqslant \operatorname{len} p_1$ and $n_3 \geqslant \operatorname{len} p_1$ and $n_5 \geqslant \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_1, n_4)$,

(ix) if $n_2 \geqslant \operatorname{len} p_1$ and $n_4 \geqslant \operatorname{len} p_1$ and $n_5 \geqslant \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_1, n_3)$, and

(x) if $n_3 \geqslant \operatorname{len} p_1$ and $n_4 \geqslant \operatorname{len} p_1$ and $n_5 \geqslant \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_1, n_2)$.

(49)(i) If $n_1 \geqslant \operatorname{len} p_1$ and $n_2 \geqslant \operatorname{len} p_1$ and $n_3 \geqslant \operatorname{len} p_1$ and $n_4 \geqslant \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_5)$,

(ii) if $n_1 \geqslant \operatorname{len} p_1$ and $n_2 \geqslant \operatorname{len} p_1$ and $n_3 \geqslant \operatorname{len} p_1$ and $n_5 \geqslant \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_4)$,

(iii) if $n_1 \geqslant \operatorname{len} p_1$ and $n_2 \geqslant \operatorname{len} p_1$ and $n_4 \geqslant \operatorname{len} p_1$ and $n_5 \geqslant \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_3)$,

(iv) if $n_1 \geqslant \operatorname{len} p_1$ and $n_3 \geqslant \operatorname{len} p_1$ and $n_4 \geqslant \operatorname{len} p_1$ and $n_5 \geqslant \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_2)$, and

(v) if $n_2 \geqslant \operatorname{len} p_1$ and $n_3 \geqslant \operatorname{len} p_1$ and $n_4 \geqslant \operatorname{len} p_1$ and $n_5 \geqslant \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_1)$.

(50) If $n_1 \geqslant \operatorname{len} p_1$ and $n_2 \geqslant \operatorname{len} p_1$ and $n_3 \geqslant \operatorname{len} p_1$ and $n_4 \geqslant \operatorname{len} p_1$ and $n_5 \geqslant \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = p_1$.

(51) $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_2, n_1, n_3, n_4, n_5)$ and $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_3, n_2, n_1, n_4, n_5)$ and $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_4, n_2, n_3, n_1, n_5)$ and $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_5, n_2, n_3, n_4, n_1)$.

(52) $\operatorname{crossover}(p_1, p_2, n_1, n_1, n_3, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_3, n_4, n_5)$ and $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_1, n_4, n_5) = \operatorname{crossover}(p_1, p_2, n_2, n_4, n_5)$ and $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_1, n_5) = \operatorname{crossover}(p_1, p_2, n_2, n_3, n_5)$ and $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_1) = \operatorname{crossover}(p_1, p_2, n_2, n_3, n_4)$.

8. Properties of 6-points Crossover

Next we state the proposition

(53) $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5, n_6)$ is a Individual of S.

Let S be a Gene-Set, let p_1, p_2 be Individual of S, and let us consider n_1, n_2, n_3, n_4, n_5, n_6. Then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5, n_6)$ is a Individual of S.

We now state four propositions:

(54)(i) $\operatorname{crossover}(p_1, p_2, 0, n_2, n_3, n_4, n_5, n_6) = \operatorname{crossover}(p_2, p_1, n_2, n_3, n_4, n_5, n_6)$,

(ii) $\operatorname{crossover}(p_1, p_2, n_1, 0, n_3, n_4, n_5, n_6) = \operatorname{crossover}(p_2, p_1, n_1, n_3, n_4, n_5, n_6)$,

(iii) $\operatorname{crossover}(p_1, p_2, n_1, n_2, 0, n_4, n_5, n_6) = \operatorname{crossover}(p_2, p_1, n_1, n_2, n_4, n_5, n_6)$,

(iv) $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, 0, n_5, n_6) = \operatorname{crossover}(p_2, p_1, n_1, n_2, n_3, n_5, n_6)$,

(v) $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, 0, n_6) = \operatorname{crossover}(p_2, p_1, n_1, n_2, n_3, n_4, n_6)$, and

(vi) $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5, 0) = \operatorname{crossover}(p_2, p_1, n_1, n_2, n_3, n_4, n_5)$.

(55)(i) If $n_1 \geqslant \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5, n_6) = \operatorname{crossover}(p_1, p_2, n_2, n_3, n_4, n_5, n_6)$,

(ii) if $n_2 \geqslant \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5, n_6) = \operatorname{crossover}(p_1, p_2, n_1, n_3, n_4, n_5, n_6)$,

(iii) if $n_3 \geqslant \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5, n_6) = \operatorname{crossover}(p_1, p_2, n_1, n_2, n_4, n_5, n_6)$,

(iv) if $n_4 \geqslant \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5, n_6) = \operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_5, n_6)$,

(v) if $n_5 \geqslant \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5, n_6) = \operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_6)$, and

(vi) if $n_6 \geqslant \operatorname{len} p_1$, then $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5, n_6) = \operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5)$.

(56)(i) $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5, n_6) = \operatorname{crossover}(p_1, p_2, n_2, n_1, n_3, n_4, n_5, n_6)$,

(ii) $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5, n_6) = \operatorname{crossover}(p_1, p_2, n_3, n_2, n_1, n_4, n_5, n_6)$,

(iii) $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5, n_6) = \operatorname{crossover}(p_1, p_2, n_4, n_2, n_3, n_1, n_5, n_6)$,

(iv) $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5, n_6) = \operatorname{crossover}(p_1, p_2, n_5, n_2, n_3, n_4, n_1, n_6)$, and

(v) $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5, n_6) = \operatorname{crossover}(p_1, p_2, n_6, n_2, n_3, n_4, n_5, n_1)$.

(57)(i) $\operatorname{crossover}(p_1, p_2, n_1, n_1, n_3, n_4, n_5, n_6) = \operatorname{crossover}(p_1, p_2, n_3, n_4, n_5, n_6)$,

(ii) $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_1, n_4, n_5, n_6) = \operatorname{crossover}(p_1, p_2, n_2, n_4, n_5, n_6)$,

(iii) $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_1, n_5, n_6) = \operatorname{crossover}(p_1, p_2, n_2, n_3, n_5, n_6)$,

(iv) $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_1, n_6) = \operatorname{crossover}(p_1, p_2, n_2, n_3, n_4, n_6)$, and

(v) $\operatorname{crossover}(p_1, p_2, n_1, n_2, n_3, n_4, n_5, n_1) = \operatorname{crossover}(p_1, p_2, n_2, n_3, n_4, n_5)$.

References

[1] Grzegorz Bancerek. The fundamental properties of natural numbers. *Formalized Mathematics*, 1(**1**):41–46, 1990.

[2] Grzegorz Bancerek. The reflection theorem. *Formalized Mathematics*, 1(**5**):973–977, 1990.

[3] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Formalized Mathematics*, 1(**1**):107–114, 1990.

[4] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.

[5] Agata Darmochwał and Yatsuka Nakamura. The topological space $\mathcal{E}^2_{\mathrm{T}}$. Arcs, line segments and special polygonal arcs. *Formalized Mathematics*, 2(**5**):617–621, 1991.

[6] Krzysztof Hryniewiecki. Basic properties of real numbers. *Formalized Mathematics*, 1(**1**):35–40, 1990.

[7] Jarosław Kotowicz. Functions and finite sequences of real numbers. *Formalized Mathematics*, 3(**2**):275–278, 1992.

[8] Andrzej Nędzusiak. σ-fields and probability. *Formalized Mathematics*, 1(**2**):401–407, 1990.

[9] Takaya Nishiyama and Yasuho Mizuhara. Binary arithmetics. *Formalized Mathematics*, 4(**1**):83–86, 1993.

[10] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.

[11] Wojciech A. Trybulec. Pigeon hole principle. *Formalized Mathematics*, 1(**3**):575–579, 1990.

[12] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(**1**):67–71, 1990.

[13] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1(**1**):73–83, 1990.

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