FORMALIZED MATHEMATICS
Volume 8, Number 1, 1999
University of Białystok

Propositional Calculus for Boolean Valued Functions. Part V

Shunichi Kobayashi
Shinshu University
Nagano

Summary. In this paper, we have proved some elementary propositional calculus formulae for Boolean valued functions.

MML Identifier: BVFUNC_9.

The terminology and notation used here have been introduced in the following articles: [3], [4], [5], [2], and [1].

In this paper Y denotes a non empty set.

We now state a number of propositions:

(1) For all elements a, b, c of $\mathrm{BVF}(Y)$ holds $(a \vee b) \wedge (b \Rightarrow c) \Subset a \vee c$.

(2) For all elements a, b of $\mathrm{BVF}(Y)$ holds $a \wedge (a \Rightarrow b) \Subset b$.

(3) For all elements a, b of $\mathrm{BVF}(Y)$ holds $(a \Rightarrow b) \wedge \neg b \Subset \neg a$.

(4) For all elements a, b of $\mathrm{BVF}(Y)$ holds $(a \vee b) \wedge \neg a \Subset b$.

(5) For all elements a, b of $\mathrm{BVF}(Y)$ holds $(a \Rightarrow b) \wedge (\neg a \Rightarrow b) \Subset b$.

(6) For all elements a, b of $\mathrm{BVF}(Y)$ holds $(a \Rightarrow b) \wedge (a \Rightarrow \neg b) \Subset \neg a$.

(7) For all elements a, b, c of $\mathrm{BVF}(Y)$ holds $a \Rightarrow b \wedge c \Subset a \Rightarrow b$.

(8) For all elements a, b, c of $\mathrm{BVF}(Y)$ holds $a \vee b \Rightarrow c \Subset a \Rightarrow c$.

(9) For all elements a, b, c of $\mathrm{BVF}(Y)$ holds $a \Rightarrow b \Subset a \wedge c \Rightarrow b$.

(10) For all elements a, b, c of $\mathrm{BVF}(Y)$ holds $a \Rightarrow b \Subset a \wedge c \Rightarrow b \wedge c$.

(11) For all elements a, b, c of $\mathrm{BVF}(Y)$ holds $a \Rightarrow b \Subset a \Rightarrow b \vee c$.

(12) For all elements a, b, c of $\mathrm{BVF}(Y)$ holds $a \Rightarrow b \Subset a \vee c \Rightarrow b \vee c$.

(13) For all elements a, b, c of $\mathrm{BVF}(Y)$ holds $a \wedge b \vee c \Subset a \vee c$.

(14) For all elements a, b, c, d of $\mathrm{BVF}(Y)$ holds $a \wedge b \vee c \wedge d \Subset a \vee c$.

ISSN 1426–2630

(15) For all elements a, b, c of BVF(Y) holds $(a \vee b) \wedge (b \Rightarrow c) \Subset a \vee c$.

(16) For all elements a, b, c of BVF(Y) holds $(a \Rightarrow b) \wedge (\neg a \Rightarrow c) \Subset b \vee c$.

(17) For all elements a, b, c of BVF(Y) holds $(a \Rightarrow c) \wedge (b \Rightarrow \neg c) \Subset \neg a \vee \neg b$.

(18) For all elements a, b, c of BVF(Y) holds $(a \vee b) \wedge (\neg a \vee c) \Subset b \vee c$.

(19) For all elements a, b, c of BVF(Y) holds $(a \Rightarrow b) \wedge (a \Rightarrow c) \Subset a \Rightarrow b \wedge c$.

(20) For all elements a, b, c, d of BVF(Y) holds $(a \Rightarrow b) \wedge (c \Rightarrow d) \Subset a \wedge c \Rightarrow b \wedge d$.

(21) For all elements a, b, c of BVF(Y) holds $(a \Rightarrow c) \wedge (b \Rightarrow c) \Subset a \vee b \Rightarrow c$.

(22) For all elements a, b, c, d of BVF(Y) holds $(a \Rightarrow b) \wedge (c \Rightarrow d) \Subset a \vee c \Rightarrow b \vee d$.

(23) For all elements a, b, c of BVF(Y) holds $(a \Rightarrow b) \wedge (a \Rightarrow c) \Subset a \Rightarrow b \vee c$.

(24) For all elements a_1, b_1, c_1, a_2, b_2, c_2 of BVF(Y) holds $(b_1 \Rightarrow b_2) \wedge (c_1 \Rightarrow c_2) \wedge (a_1 \vee b_1 \vee c_1) \wedge \neg(a_2 \wedge b_2) \wedge \neg(a_2 \wedge c_2) \Subset a_2 \Rightarrow a_1$.

(25) For all elements a_1, b_1, c_1, a_2, b_2, c_2 of BVF(Y) holds $(a_1 \Rightarrow a_2) \wedge (b_1 \Rightarrow b_2) \wedge (c_1 \Rightarrow c_2) \wedge (a_1 \vee b_1 \vee c_1) \wedge \neg(a_2 \wedge b_2) \wedge \neg(a_2 \wedge c_2) \wedge \neg(b_2 \wedge c_2) \Subset (a_2 \Rightarrow a_1) \wedge (b_2 \Rightarrow b_1) \wedge (c_2 \Rightarrow c_1)$.

(26) For all elements a_1, b_1, a_2, b_2 of BVF(Y) holds $(a_1 \Rightarrow a_2) \wedge (b_1 \Rightarrow b_2) \wedge \neg(a_2 \wedge b_2) \Rightarrow \neg(a_1 \wedge b_1) = \mathit{true}(Y)$.

(27) For all elements a_1, b_1, c_1, a_2, b_2, c_2 of BVF(Y) holds $(a_1 \Rightarrow a_2) \wedge (b_1 \Rightarrow b_2) \wedge (c_1 \Rightarrow c_2) \wedge \neg(a_2 \wedge b_2) \wedge \neg(a_2 \wedge c_2) \wedge \neg(b_2 \wedge c_2) \Subset \neg(a_1 \wedge b_1) \wedge \neg(a_1 \wedge c_1) \wedge \neg(b_1 \wedge c_1)$.

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Received May 5, 1999