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Predicate Calculus for Boolean Valued Functions. Part I

Shunichi Kobayashi
Shinshu University
Nagano

Yatsuka Nakamura
Shinshu University
Nagano

Summary. In this paper, we have proved some elementary predicate calculus formulae containing the quantifiers of Boolean valued functions with respect to partitions. Such a theory is an analogy of usual predicate logic.

MML Identifier: BVFUNC_3.

The terminology and notation used here are introduced in the following articles: [1], [2], [3], and [4].

For simplicity, we adopt the following convention: Y denotes a non empty set, G denotes a subset of PARTITIONS(Y), a, b, c, u denote elements of BVF(Y), and P_1 denotes a partition of Y.

The following propositions are true:

(1) $a \Rightarrow b \Subset \forall_{a,P_1} G \Rightarrow \exists_{b,P_1} G$.

(2) $\forall_{a,P_1} G \wedge \forall_{b,P_1} G \Subset a \wedge b$.

(3) $a \wedge b \Subset \exists_{a,P_1} G \wedge \exists_{b,P_1} G$.

(4) $\neg(\forall_{a,P_1} G \wedge \forall_{b,P_1} G) = \exists_{\neg a,P_1} G \vee \exists_{\neg b,P_1} G$.

(5) $\neg(\exists_{a,P_1} G \wedge \exists_{b,P_1} G) = \forall_{\neg a,P_1} G \vee \forall_{\neg b,P_1} G$.

(6) $\forall_{a,P_1} G \vee \forall_{b,P_1} G \Subset a \vee b$.

(7) $a \vee b \Subset \exists_{a,P_1} G \vee \exists_{b,P_1} G$.

(8) $a \oplus b \Subset \neg(\exists_{\neg a,P_1} G \oplus \exists_{b,P_1} G) \vee \neg(\exists_{a,P_1} G \oplus \exists_{\neg b,P_1} G)$.

(9) $\forall_{a \vee b,P_1} G \Subset \forall_{a,P_1} G \vee \exists_{b,P_1} G$.

(10) $\forall_{a \vee b,P_1} G \Subset \exists_{a,P_1} G \vee \forall_{b,P_1} G$.

(11) $\forall_{a \vee b,P_1} G \Subset \exists_{a,P_1} G \vee \exists_{b,P_1} G$.

(12) $\exists_{a,P_1} G \wedge \forall_{b,P_1} G \Subset \exists_{a \wedge b,P_1} G$.

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(13) $\forall_{a,P_1}G \wedge \exists_{b,P_1}G \Subset \exists_{a\wedge b,P_1}G$.

(14) $\forall_{a\Rightarrow b,P_1}G \Subset \forall_{a,P_1}G \Rightarrow \exists_{b,P_1}G$.

(15) $\forall_{a\Rightarrow b,P_1}G \Subset \exists_{a,P_1}G \Rightarrow \exists_{b,P_1}G$.

(16) $\exists_{a,P_1}G \Rightarrow \forall_{b,P_1}G \Subset \forall_{a\Rightarrow b,P_1}G$.

(17) $a \Rightarrow b \Subset a \Rightarrow \exists_{b,P_1}G$.

(18) $a \Rightarrow b \Subset \forall_{a,P_1}G \Rightarrow b$.

(19) $\exists_{a\Rightarrow b,P_1}G \Subset \forall_{a,P_1}G \Rightarrow \exists_{b,P_1}G$.

(20) $\forall_{a,P_1}G \Subset \exists_{b,P_1}G \Rightarrow \exists_{a\wedge b,P_1}G$.

(21) If u is independent of P_1, G, then $\exists_{u\Rightarrow a,P_1}G \Subset u \Rightarrow \exists_{a,P_1}G$.

(22) If u is independent of P_1, G, then $\exists_{a\Rightarrow u,P_1}G \Subset \forall_{a,P_1}G \Rightarrow u$.

(23) $\forall_{a,P_1}G \Rightarrow \exists_{b,P_1}G = \exists_{a\Rightarrow b,P_1}G$.

(24) $\forall_{a,P_1}G \Rightarrow \forall_{b,P_1}G \Subset \forall_{a,P_1}G \Rightarrow \exists_{b,P_1}G$.

(25) $\exists_{a,P_1}G \Rightarrow \exists_{b,P_1}G \Subset \forall_{a,P_1}G \Rightarrow \exists_{b,P_1}G$.

(26) $\forall_{a\Rightarrow b,P_1}G = \forall_{\neg a\vee b,P_1}G$.

(27) If G is a coordinate and $P_1 \in G$, then $\forall_{a\Rightarrow b,P_1}G = \neg\exists_{a\wedge\neg b,P_1}G$.

(28) $\exists_{a,P_1}G \Subset \neg(\forall_{a\Rightarrow b,P_1}G \wedge \forall_{a\Rightarrow\neg b,P_1}G)$.

(29) $\exists_{a,P_1}G \Subset \neg(\neg\exists_{a\wedge b,P_1}G \wedge \neg\exists_{a\wedge\neg b,P_1}G)$.

(30) $\exists_{a,P_1}G \wedge \forall_{a\Rightarrow b,P_1}G \Subset \exists_{a\wedge b,P_1}G$.

(31) $\exists_{a,P_1}G \wedge \neg\exists_{a\wedge b,P_1}G \Subset \neg\forall_{a\Rightarrow b,P_1}G$.

(32) $\forall_{a\Rightarrow c,P_1}G \wedge \forall_{c\Rightarrow b,P_1}G \Subset \forall_{a\Rightarrow b,P_1}G$.

(33) $\forall_{c\Rightarrow b,P_1}G \wedge \exists_{a\wedge c,P_1}G \Subset \exists_{a\wedge b,P_1}G$.

(34) $\forall_{b\Rightarrow\neg c,P_1}G \wedge \forall_{a\Rightarrow c,P_1}G \Subset \forall_{a\Rightarrow\neg b,P_1}G$.

(35) $\forall_{b\Rightarrow c,P_1}G \wedge \forall_{a\Rightarrow\neg c,P_1}G \Subset \forall_{a\Rightarrow\neg b,P_1}G$.

(36) $\forall_{b\Rightarrow\neg c,P_1}G \wedge \exists_{a\wedge c,P_1}G \Subset \exists_{a\wedge\neg b,P_1}G$.

(37) $\forall_{b\Rightarrow c,P_1}G \wedge \exists_{a\wedge\neg c,P_1}G \Subset \exists_{a\wedge\neg b,P_1}G$.

(38) $\exists_{c,P_1}G \wedge \forall_{c\Rightarrow b,P_1}G \wedge \forall_{c\Rightarrow a,P_1}G \Subset \exists_{a\wedge b,P_1}G$.

(39) $\forall_{b\Rightarrow c,P_1}G \wedge \forall_{c\Rightarrow\neg a,P_1}G \Subset \forall_{a\Rightarrow\neg b,P_1}G$.

(40) $\exists_{b,P_1}G \wedge \forall_{b\Rightarrow c,P_1}G \wedge \forall_{c\Rightarrow a,P_1}G \Subset \exists_{a\wedge b,P_1}G$.

(41) $\exists_{c,P_1}G \wedge \forall_{b\Rightarrow\neg c,P_1}G \wedge \forall_{c\Rightarrow a,P_1}G \Subset \exists_{a\wedge\neg b,P_1}G$.

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