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Some Basic Properties of Many Sorted Sets

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The notation and terminology used here are introduced in the following papers: [11], [12], [5], [13], [2], [3], [4], [6], [1], [10], [9], [8], and [7].

1. Preliminaries

For simplicity we follow a convention: i will be arbitrary, I will be a set, f will be a function, x, x_1, x_2, y, A, B, X, Y, Z will be many sorted sets indexed by I, J will be a non empty set, and N_1 will be a many sorted set indexed by J.

We now state three propositions:

(1) For every set X and for every many sorted set M indexed by I such that $i \in I$ holds $\mathrm{dom}(M +\cdot (i \dot{\longmapsto} X)) = I$.

(2) If $f = \emptyset$, then f is a many sorted set indexed by $\emptyset$.

(3) If I is non empty, then there exists no X which is empty yielding and non-empty.

2. Singelton and unordered pairs

Let us consider I, A. The functor $\{A\}$ yielding a many sorted set indexed by I is defined as follows:

(Def.1) For every i such that $i \in I$ holds $\{A\}(i) = \{A(i)\}$.

Let us consider I, A. Observe that $\{A\}$ is non-empty and locally-finite.

Let us consider I, A, B. The functor $\{A, B\}$ yields a many sorted set indexed by I and is defined as follows:

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(Def.2) For every i such that $i \in I$ holds $\{A, B\}(i) = \{A(i), B(i)\}$.

Let us consider I, A, B. One can verify that $\{A, B\}$ is non-empty and locally-finite.

We now state a number of propositions:

(4) $X = \{y\}$ iff for every x holds $x \in X$ iff $x = y$.

(5) If for every x holds $x \in X$ iff $x = x_1$ or $x = x_2$, then $X = \{x_1, x_2\}$.

(6) If $X = \{x_1, x_2\}$, then for every x such that $x = x_1$ or $x = x_2$ holds $x \in X$.

(7) $\{N_1\} \neq \emptyset_I$.

(8) If $x \in \{A\}$, then $x = A$.

(9) $x \in \{x\}$.

(10) If $x = A$ or $x = B$, then $x \in \{A, B\}$.

(11) $\{A\} \cup \{B\} = \{A, B\}$.

(12) $\{x, x\} = \{x\}$.

(13) $\{A, B\} = \{B, A\}$.

(14) If $\{A\} \subseteq \{B\}$, then $A = B$.

(15) If $\{x\} = \{y\}$, then $x = y$.

(16) If $\{x\} = \{A, B\}$, then $x = A$ and $x = B$.

(17) If $\{x\} = \{A, B\}$, then $A = B$.

(18) $\{x\} \subseteq \{x, y\}$ and $\{y\} \subseteq \{x, y\}$.

(19) If $\{x\} \cup \{y\} = \{x\}$ or $\{x\} \cup \{y\} = \{y\}$, then $x = y$.

(20) $\{x\} \cup \{x, y\} = \{x, y\}$.

(21) $\{y\} \cup \{x, y\} = \{x, y\}$.

(22) If I is non empty and $\{x\} \cap \{y\} = \emptyset_I$, then $x \neq y$.

(23) If $\{x\} \cap \{y\} = \{x\}$ or $\{x\} \cap \{y\} = \{y\}$, then $x = y$.

(24) $\{x\} \cap \{x, y\} = \{x\}$ and $\{y\} \cap \{x, y\} = \{y\}$.

(25) If I is non empty and $\{x\} \setminus \{y\} = \{x\}$, then $x \neq y$.

(26) If $\{x\} \setminus \{y\} = \emptyset_I$, then $x = y$.

(27) $\{x\} \setminus \{x, y\} = \emptyset_I$ and $\{y\} \setminus \{x, y\} = \emptyset_I$.

(28) If $\{x\} \subseteq \{y\}$, then $\{x\} = \{y\}$.

(29) If $\{x, y\} \subseteq \{A\}$, then $x = A$ and $y = A$.

(30) If $\{x, y\} \subseteq \{A\}$, then $\{x, y\} = \{A\}$.

(31) $2^{\{x\}} = \{\emptyset_I, \{x\}\}$.

(32) $\{A\} \subseteq 2^A$.

(33) $\bigcup\{x\} = x$.

(34) $\bigcup\{\{x\}, \{y\}\} = \{x, y\}$.

(35) $\bigcup\{A, B\} = A \cup B$.

(36) $\{x\} \subseteq X$ iff $x \in X$.

(37) $\{x_1, x_2\} \subseteq X$ iff $x_1 \in X$ and $x_2 \in X$.

(38) If $A = \emptyset_I$ or $A = \{x_1\}$ or $A = \{x_2\}$ or $A = \{x_1, x_2\}$, then $A \subseteq \{x_1, x_2\}$.

3. Sum of unordered pairs (or a singelton) and a set

One can prove the following propositions:

(39) If $x \in A$ or $x = B$, then $x \in A \cup \{B\}$.

(40) $A \cup \{x\} \subseteq B$ iff $x \in B$ and $A \subseteq B$.

(41) If $\{x\} \cup X = X$, then $x \in X$.

(42) If $x \in X$, then $\{x\} \cup X = X$.

(43) $\{x, y\} \cup A = A$ iff $x \in A$ and $y \in A$.

(44) If I is non empty, then $\{x\} \cup X \neq \emptyset_I$.

(45) If I is non empty, then $\{x, y\} \cup X \neq \emptyset_I$.

4. Intersection of unordered pairs (or a singelton) and a set

We now state several propositions:

(46) If $X \cap \{x\} = \{x\}$, then $x \in X$.

(47) If $x \in X$, then $X \cap \{x\} = \{x\}$.

(48) $x \in X$ and $y \in X$ iff $\{x, y\} \cap X = \{x, y\}$.

(49) If I is non empty and $\{x\} \cap X = \emptyset_I$, then $x \notin X$.

(50) If I is non empty and $\{x, y\} \cap X = \emptyset_I$, then $x \notin X$ and $y \notin X$.

5. Difference of unordered pairs (or a singelton) and a set

The following propositions are true:

(51) If $y \in X \setminus \{x\}$, then $y \in X$.

(52) If I is non empty and $y \in X \setminus \{x\}$, then $y \neq x$.

(53) If I is non empty and $X \setminus \{x\} = X$, then $x \notin X$.

(54) If I is non empty and $\{x\} \setminus X = \{x\}$, then $x \notin X$.

(55) $\{x\} \setminus X = \emptyset_I$ iff $x \in X$.

(56) If I is non empty and $\{x, y\} \setminus X = \{x\}$, then $x \notin X$.

(57) If I is non empty and $\{x, y\} \setminus X = \{y\}$, then $y \notin X$.

(58) If I is non empty and $\{x, y\} \setminus X = \{x, y\}$, then $x \notin X$ and $y \notin X$.

(59) $\{x, y\} \setminus X = \emptyset_I$ iff $x \in X$ and $y \in X$.

(60) If $X = \emptyset_I$ or $X = \{x\}$ or $X = \{y\}$ or $X = \{x, y\}$, then $X \setminus \{x, y\} = \emptyset_I$.

6. Cartesian product

One can prove the following propositions:

(61) If $X = \emptyset_I$ or $Y = \emptyset_I$, then $[\![X, Y]\!] = \emptyset_I$.

(62) If X is non-empty and Y is non-empty and $[\![X, Y]\!] = [\![Y, X]\!]$, then $X = Y$.

(63) If $[\![X, X]\!] = [\![Y, Y]\!]$, then $X = Y$.

(64) If Z is non-empty and if $[\![X, Z]\!] \subseteq [\![Y, Z]\!]$ or $[\![Z, X]\!] \subseteq [\![Z, Y]\!]$, then $X \subseteq Y$.

(65) If $X \subseteq Y$, then $[\![X, Z]\!] \subseteq [\![Y, Z]\!]$ and $[\![Z, X]\!] \subseteq [\![Z, Y]\!]$.

(66) If $x_1 \subseteq A$ and $x_2 \subseteq B$, then $[\![x_1, x_2]\!] \subseteq [\![A, B]\!]$.

(67) $[\![X \cup Y, Z]\!] = [\![X, Z]\!] \cup [\![Y, Z]\!]$ and $[\![Z, X \cup Y]\!] = [\![Z, X]\!] \cup [\![Z, Y]\!]$.

(68) $[\![x_1 \cup x_2, A \cup B]\!] = [\![x_1, A]\!] \cup [\![x_1, B]\!] \cup [\![x_2, A]\!] \cup [\![x_2, B]\!]$.

(69) $[\![X \cap Y, Z]\!] = [\![X, Z]\!] \cap [\![Y, Z]\!]$ and $[\![Z, X \cap Y]\!] = [\![Z, X]\!] \cap [\![Z, Y]\!]$.

(70) $[\![x_1 \cap x_2, A \cap B]\!] = [\![x_1, A]\!] \cap [\![x_2, B]\!]$.

(71) If $A \subseteq X$ and $B \subseteq Y$, then $[\![A, Y]\!] \cap [\![X, B]\!] = [\![A, B]\!]$.

(72) $[\![X \setminus Y, Z]\!] = [\![X, Z]\!] \setminus [\![Y, Z]\!]$ and $[\![Z, X \setminus Y]\!] = [\![Z, X]\!] \setminus [\![Z, Y]\!]$.

(73) $[\![x_1, x_2]\!] \setminus [\![A, B]\!] = [\![x_1 \setminus A, x_2]\!] \cup [\![x_1, x_2 \setminus B]\!]$.

(74) If $x_1 \cap x_2 = \emptyset_I$ or $A \cap B = \emptyset_I$, then $[\![x_1, A]\!] \cap [\![x_2, B]\!] = \emptyset_I$.

(75) If X is non-empty, then $[\![\{x\}, X]\!]$ is non-empty and $[\![X, \{x\}]\!]$ is non-empty.

(76) $[\![\{x, y\}, X]\!] = [\![\{x\}, X]\!] \cup [\![\{y\}, X]\!]$ and $[\![X, \{x, y\}]\!] = [\![X, \{x\}]\!] \cup [\![X, \{y\}]\!]$.

(77) If x_1 is non-empty and A is non-empty and $[\![x_1, A]\!] = [\![x_2, B]\!]$, then $x_1 = x_2$ and $A = B$.

(78) If $X \subseteq [\![X, Y]\!]$ or $X \subseteq [\![Y, X]\!]$, then $X = \emptyset_I$.

(79) If $A \in [\![x, y]\!]$ and $A \in [\![X, Y]\!]$, then $A \in [\![x \cap X, y \cap Y]\!]$.

(80) If $[\![x, X]\!] \subseteq [\![y, Y]\!]$ and $[\![x, X]\!]$ is non-empty, then $x \subseteq y$ and $X \subseteq Y$.

(81) If $A \subseteq X$, then $[\![A, A]\!] \subseteq [\![X, X]\!]$.

(82) If $X \cap Y = \emptyset_I$, then $[\![X, Y]\!] \cap [\![Y, X]\!] = \emptyset_I$.

(83) If A is non-empty and if $[\![A, B]\!] \subseteq [\![X, Y]\!]$ or $[\![B, A]\!] \subseteq [\![Y, X]\!]$, then $B \subseteq Y$.

(84) If $x \subseteq [\![A, B]\!]$ and $y \subseteq [\![X, Y]\!]$, then $x \cup y \subseteq [\![A \cup X, B \cup Y]\!]$.

References

[1] Czesław Byliński. A classical first order language. *Formalized Mathematics*, 1(**4**):669–676, 1990.

[2] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.

[3] Czesław Byliński. Functions from a set to a set. *Formalized Mathematics*, 1(**1**):153–164, 1990.

[4] Czesław Byliński. The modification of a function by a function and the iteration of the composition of a function. *Formalized Mathematics*, 1(**3**):521–527, 1990.
[5] Czesław Byliński. Some basic properties of sets. *Formalized Mathematics*, 1(**1**):47–53, 1990.
[6] Agata Darmochwał. Finite sets. *Formalized Mathematics*, 1(**1**):165–167, 1990.
[7] Artur Korniłowicz. Definitions and basic properties of boolean & union of many sorted sets. *Formalized Mathematics*, 5(**2**):279–281, 1996.
[8] Beata Madras. Products of many sorted algebras. *Formalized Mathematics*, 5(**1**):55–60, 1996.
[9] Yatsuka Nakamura, Piotr Rudnicki, Andrzej Trybulec, and Pauline N. Kawamoto. Preliminaries to circuits, I. *Formalized Mathematics*, 5(**2**):167–172, 1996.
[10] Andrzej Trybulec. Many-sorted sets. *Formalized Mathematics*, 4(**1**):15–22, 1993.
[11] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.
[12] Zinaida Trybulec and Halina Święczkowska. Boolean properties of sets. *Formalized Mathematics*, 1(**1**):17–23, 1990.
[13] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1(**1**):73–83, 1990.

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