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On the Geometry of a Go-Board

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The articles [15], [17], [7], [1], [14], [16], [12], [4], [2], [8], [9], [13], [18], [3], [5], [6], [10], and [11] provide the notation and terminology for this paper.

For simplicity we follow the rules: i, j, n will be natural numbers, r, s, r_1, s_1, r_2, s_2 will be real numbers, p will be a point of $\mathcal{E}^2_{\rm T}$, G will be a Go-board, M will be a metric space, and u will be a point of $\mathcal{E}^2$.

One can prove the following propositions:

(4)[1] For every metric space M and for every point u of M such that $r > 0$ holds $u \in \mathrm{Ball}(u, r)$.

(6)[2] For every subset B of the carrier of $\mathcal{E}^n_{\rm T}$ and for every point u of $\mathcal{E}^n$ such that $B = \mathrm{Ball}(u, r)$ holds B is open.

(7) Let M be a metric space, and let u be a point of M, and let P be a subset of the carrier of $M_{\rm top}$. Then $u \in \mathrm{Int}\, P$ if and only if there exists r such that $r > 0$ and $\mathrm{Ball}(u, r) \subseteq P$.

(8) Let u be a point of $\mathcal{E}^n$ and let P be a subset of the carrier of $\mathcal{E}^n_{\rm T}$. Then $u \in \mathrm{Int}\, P$ if and only if there exists r such that $r > 0$ and $\mathrm{Ball}(u, r) \subseteq P$.

(9) For all points u, v of $\mathcal{E}^2$ such that $u = [r_1, s_1]$ and $v = [r_2, s_2]$ holds $\rho(u, v) = \sqrt{(r_1 - r_2)^{\mathbf{2}} + (s_1 - s_2)^{\mathbf{2}}}$.

(10) For every point u of $\mathcal{E}^2$ such that $u = [r, s]$ holds if $0 \leq r_2$ and $r_2 < r_1$, then $[r + r_2, s] \in \mathrm{Ball}(u, r_1)$.

(11) For every point u of $\mathcal{E}^2$ such that $u = [r, s]$ holds if $0 \leq s_2$ and $s_2 < s_1$, then $[r, s + s_2] \in \mathrm{Ball}(u, s_1)$.

(12) For every point u of $\mathcal{E}^2$ such that $u = [r, s]$ holds if $0 \leq r_2$ and $r_2 < r_1$, then $[r - r_2, s] \in \mathrm{Ball}(u, r_1)$.

[1]The propositions (1)–(3) have been removed.
[2]The proposition (5) has been removed.

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(13) For every point u of $\mathcal{E}^2$ such that $u = [r, s]$ holds if $0 \leq s_2$ and $s_2 < s_1$, then $[r, s - s_2] \in \mathrm{Ball}(u, s_1)$.

(14) If $1 \leq i$ and $i < \mathrm{len}\, G$ and $1 \leq j$ and $j < \mathrm{width}\, G$, then $G_{i,j} + G_{i+1,j+1} = G_{i,j+1} + G_{i+1,j}$.

(15) $\mathrm{Int}\, \mathrm{vstrip}(G, 0) = \{[r, s] : r < (G_{1,1})_{\mathbf{1}}\}$.

(16) $\mathrm{Int}\, \mathrm{vstrip}(G, \mathrm{len}\, G) = \{[r, s] : (G_{\mathrm{len}\, G,1})_{\mathbf{1}} < r\}$.

(17) If $1 \leq i$ and $i < \mathrm{len}\, G$, then $\mathrm{Int}\, \mathrm{vstrip}(G, i) = \{[r, s] : (G_{i,1})_{\mathbf{1}} < r \wedge r < (G_{i+1,1})_{\mathbf{1}}\}$.

(18) $\mathrm{Int}\, \mathrm{hstrip}(G, 0) = \{[r, s] : s < (G_{1,1})_{\mathbf{2}}\}$.

(19) $\mathrm{Int}\, \mathrm{hstrip}(G, \mathrm{width}\, G) = \{[r, s] : (G_{1,\mathrm{width}\, G})_{\mathbf{2}} < s\}$.

(20) If $1 \leq j$ and $j < \mathrm{width}\, G$, then $\mathrm{Int}\, \mathrm{hstrip}(G, j) = \{[r, s] : (G_{1,j})_{\mathbf{2}} < s \wedge s < (G_{1,j+1})_{\mathbf{2}}\}$.

(21) $\mathrm{Int}\, \mathrm{cell}(G, 0, 0) = \{[r, s] : r < (G_{1,1})_{\mathbf{1}} \wedge s < (G_{1,1})_{\mathbf{2}}\}$.

(22) $\mathrm{Int}\, \mathrm{cell}(G, 0, \mathrm{width}\, G) = \{[r, s] : r < (G_{1,1})_{\mathbf{1}} \wedge (G_{1,\mathrm{width}\, G})_{\mathbf{2}} < s\}$.

(23) If $1 \leq j$ and $j < \mathrm{width}\, G$, then $\mathrm{Int}\, \mathrm{cell}(G, 0, j) = \{[r, s] : r < (G_{1,1})_{\mathbf{1}} \wedge (G_{1,j})_{\mathbf{2}} < s \wedge s < (G_{1,j+1})_{\mathbf{2}}\}$.

(24) $\mathrm{Int}\, \mathrm{cell}(G, \mathrm{len}\, G, 0) = \{[r, s] : (G_{\mathrm{len}\, G,1})_{\mathbf{1}} < r \wedge s < (G_{1,1})_{\mathbf{2}}\}$.

(25) $\mathrm{Int}\, \mathrm{cell}(G, \mathrm{len}\, G, \mathrm{width}\, G) = \{[r, s] : (G_{\mathrm{len}\, G,1})_{\mathbf{1}} < r \wedge (G_{1,\mathrm{width}\, G})_{\mathbf{2}} < s\}$.

(26) If $1 \leq j$ and $j < \mathrm{width}\, G$, then $\mathrm{Int}\, \mathrm{cell}(G, \mathrm{len}\, G, j) = \{[r, s] : (G_{\mathrm{len}\, G,1})_{\mathbf{1}} < r \wedge (G_{1,j})_{\mathbf{2}} < s \wedge s < (G_{1,j+1})_{\mathbf{2}}\}$.

(27) If $1 \leq i$ and $i < \mathrm{len}\, G$, then $\mathrm{Int}\, \mathrm{cell}(G, i, 0) = \{[r, s] : (G_{i,1})_{\mathbf{1}} < r \wedge r < (G_{i+1,1})_{\mathbf{1}} \wedge s < (G_{1,1})_{\mathbf{2}}\}$.

(28) If $1 \leq i$ and $i < \mathrm{len}\, G$, then $\mathrm{Int}\, \mathrm{cell}(G, i, \mathrm{width}\, G) = \{[r, s] : (G_{i,1})_{\mathbf{1}} < r \wedge r < (G_{i+1,1})_{\mathbf{1}} \wedge (G_{1,\mathrm{width}\, G})_{\mathbf{2}} < s\}$.

(29) If $1 \leq i$ and $i < \mathrm{len}\, G$ and $1 \leq j$ and $j < \mathrm{width}\, G$, then $\mathrm{Int}\, \mathrm{cell}(G, i, j) = \{[r, s] : (G_{i,1})_{\mathbf{1}} < r \wedge r < (G_{i+1,1})_{\mathbf{1}} \wedge (G_{1,j})_{\mathbf{2}} < s \wedge s < (G_{1,j+1})_{\mathbf{2}}\}$.

(30) If $1 \leq j$ and $j \leq \mathrm{width}\, G$ and $p \in \mathrm{Int}\, \mathrm{hstrip}(G, j)$, then $p_{\mathbf{2}} > (G_{1,j})_{\mathbf{2}}$.

(31) If $j < \mathrm{width}\, G$ and $p \in \mathrm{Int}\, \mathrm{hstrip}(G, j)$, then $p_{\mathbf{2}} < (G_{1,j+1})_{\mathbf{2}}$.

(32) If $1 \leq i$ and $i \leq \mathrm{len}\, G$ and $p \in \mathrm{Int}\, \mathrm{vstrip}(G, i)$, then $p_{\mathbf{1}} > (G_{i,1})_{\mathbf{1}}$.

(33) If $i < \mathrm{len}\, G$ and $p \in \mathrm{Int}\, \mathrm{vstrip}(G, i)$, then $p_{\mathbf{1}} < (G_{i+1,1})_{\mathbf{1}}$.

(34) If $1 \leq i$ and $i + 1 \leq \mathrm{len}\, G$ and $1 \leq j$ and $j + 1 \leq \mathrm{width}\, G$, then $\frac{1}{2} \cdot (G_{i,j} + G_{i+1,j+1}) \in \mathrm{Int}\, \mathrm{cell}(G, i, j)$.

(35) If $1 \leq i$ and $i + 1 \leq \mathrm{len}\, G$, then $\frac{1}{2} \cdot (G_{i,\mathrm{width}\, G} + G_{i+1,\mathrm{width}\, G}) + [0, 1] \in \mathrm{Int}\, \mathrm{cell}(G, i, \mathrm{width}\, G)$.

(36) If $1 \leq i$ and $i+1 \leq \mathrm{len}\, G$, then $\frac{1}{2} \cdot (G_{i,1} + G_{i+1,1}) - [0, 1] \in \mathrm{Int}\, \mathrm{cell}(G, i, 0)$.

(37) If $1 \leq j$ and $j + 1 \leq \mathrm{width}\, G$, then $\frac{1}{2} \cdot (G_{\mathrm{len}\, G,j} + G_{\mathrm{len}\, G,j+1}) + [1, 0] \in \mathrm{Int}\, \mathrm{cell}(G, \mathrm{len}\, G, j)$.

(38) If $1 \leq j$ and $j + 1 \leq \mathrm{width}\, G$, then $\frac{1}{2} \cdot (G_{1,j} + G_{1,j+1}) - [1, 0] \in \mathrm{Int}\, \mathrm{cell}(G, 0, j)$.

(39) $G_{1,1} - [1, 1] \in \mathrm{Int}\, \mathrm{cell}(G, 0, 0)$.

(40) $G_{\operatorname{len} G, \operatorname{width} G} + [1,1] \in \operatorname{Int} \operatorname{cell}(G, \operatorname{len} G, \operatorname{width} G)$.

(41) $G_{1, \operatorname{width} G} + [-1,1] \in \operatorname{Int} \operatorname{cell}(G, 0, \operatorname{width} G)$.

(42) $G_{\operatorname{len} G, 1} + [1,-1] \in \operatorname{Int} \operatorname{cell}(G, \operatorname{len} G, 0)$.

(43) If $1 \leq i$ and $i < \operatorname{len} G$ and $1 \leq j$ and $j < \operatorname{width} G$, then $\mathcal{L}(\frac{1}{2} \cdot (G_{i,j} + G_{i+1,j+1}), \frac{1}{2} \cdot (G_{i,j} + G_{i,j+1})) \subseteq \operatorname{Int} \operatorname{cell}(G, i, j) \cup \{\frac{1}{2} \cdot (G_{i,j} + G_{i,j+1})\}$.

(44) Suppose $1 \leq i$ and $i < \operatorname{len} G$ and $1 \leq j$ and $j < \operatorname{width} G$. Then $\mathcal{L}(\frac{1}{2} \cdot (G_{i,j} + G_{i+1,j+1}), \frac{1}{2} \cdot (G_{i,j+1} + G_{i+1,j+1})) \subseteq \operatorname{Int} \operatorname{cell}(G, i, j) \cup \{\frac{1}{2} \cdot (G_{i,j+1} + G_{i+1,j+1})\}$.

(45) Suppose $1 \leq i$ and $i < \operatorname{len} G$ and $1 \leq j$ and $j < \operatorname{width} G$. Then $\mathcal{L}(\frac{1}{2} \cdot (G_{i,j} + G_{i+1,j+1}), \frac{1}{2} \cdot (G_{i+1,j} + G_{i+1,j+1})) \subseteq \operatorname{Int} \operatorname{cell}(G, i, j) \cup \{\frac{1}{2} \cdot (G_{i+1,j} + G_{i+1,j+1})\}$.

(46) If $1 \leq i$ and $i < \operatorname{len} G$ and $1 \leq j$ and $j < \operatorname{width} G$, then $\mathcal{L}(\frac{1}{2} \cdot (G_{i,j} + G_{i+1,j+1}), \frac{1}{2} \cdot (G_{i,j} + G_{i+1,j})) \subseteq \operatorname{Int} \operatorname{cell}(G, i, j) \cup \{\frac{1}{2} \cdot (G_{i,j} + G_{i+1,j})\}$.

(47) If $1 \leq j$ and $j < \operatorname{width} G$, then $\mathcal{L}(\frac{1}{2} \cdot (G_{1,j} + G_{1,j+1}) - [1,0], \frac{1}{2} \cdot (G_{1,j} + G_{1,j+1})) \subseteq \operatorname{Int} \operatorname{cell}(G, 0, j) \cup \{\frac{1}{2} \cdot (G_{1,j} + G_{1,j+1})\}$.

(48) If $1 \leq j$ and $j < \operatorname{width} G$, then $\mathcal{L}(\frac{1}{2} \cdot (G_{\operatorname{len} G,j} + G_{\operatorname{len} G,j+1}) + [1,0], \frac{1}{2} \cdot (G_{\operatorname{len} G,j} + G_{\operatorname{len} G,j+1})) \subseteq \operatorname{Int} \operatorname{cell}(G, \operatorname{len} G, j) \cup \{\frac{1}{2} \cdot (G_{\operatorname{len} G,j} + G_{\operatorname{len} G,j+1})\}$.

(49) If $1 \leq i$ and $i < \operatorname{len} G$, then $\mathcal{L}(\frac{1}{2} \cdot (G_{i,1} + G_{i+1,1}) - [0,1], \frac{1}{2} \cdot (G_{i,1} + G_{i+1,1})) \subseteq \operatorname{Int} \operatorname{cell}(G, i, 0) \cup \{\frac{1}{2} \cdot (G_{i,1} + G_{i+1,1})\}$.

(50) If $1 \leq i$ and $i < \operatorname{len} G$, then $\mathcal{L}(\frac{1}{2} \cdot (G_{i,\operatorname{width} G} + G_{i+1,\operatorname{width} G}) + [0,1], \frac{1}{2} \cdot (G_{i,\operatorname{width} G} + G_{i+1,\operatorname{width} G})) \subseteq \operatorname{Int} \operatorname{cell}(G, i, \operatorname{width} G) \cup \{\frac{1}{2} \cdot (G_{i,\operatorname{width} G} + G_{i+1,\operatorname{width} G})\}$.

(51) If $1 \leq j$ and $j < \operatorname{width} G$, then $\mathcal{L}(\frac{1}{2} \cdot (G_{1,j} + G_{1,j+1}) - [1,0], G_{1,j} - [1,0]) \subseteq \operatorname{Int} \operatorname{cell}(G, 0, j) \cup \{G_{1,j} - [1,0]\}$.

(52) If $1 \leq j$ and $j < \operatorname{width} G$, then $\mathcal{L}(\frac{1}{2} \cdot (G_{1,j} + G_{1,j+1}) - [1,0], G_{1,j+1} - [1,0]) \subseteq \operatorname{Int} \operatorname{cell}(G, 0, j) \cup \{G_{1,j+1} - [1,0]\}$.

(53) If $1 \leq j$ and $j < \operatorname{width} G$, then $\mathcal{L}(\frac{1}{2} \cdot (G_{\operatorname{len} G,j} + G_{\operatorname{len} G,j+1}) + [1,0], G_{\operatorname{len} G,j} + [1,0]) \subseteq \operatorname{Int} \operatorname{cell}(G, \operatorname{len} G, j) \cup \{G_{\operatorname{len} G,j} + [1,0]\}$.

(54) If $1 \leq j$ and $j < \operatorname{width} G$, then $\mathcal{L}(\frac{1}{2} \cdot (G_{\operatorname{len} G,j} + G_{\operatorname{len} G,j+1}) + [1,0], G_{\operatorname{len} G,j+1} + [1,0]) \subseteq \operatorname{Int} \operatorname{cell}(G, \operatorname{len} G, j) \cup \{G_{\operatorname{len} G,j+1} + [1,0]\}$.

(55) If $1 \leq i$ and $i < \operatorname{len} G$, then $\mathcal{L}(\frac{1}{2} \cdot (G_{i,1} + G_{i+1,1}) - [0,1], G_{i,1} - [0,1]) \subseteq \operatorname{Int} \operatorname{cell}(G, i, 0) \cup \{G_{i,1} - [0,1]\}$.

(56) If $1 \leq i$ and $i < \operatorname{len} G$, then $\mathcal{L}(\frac{1}{2} \cdot (G_{i,1} + G_{i+1,1}) - [0,1], G_{i+1,1} - [0,1]) \subseteq \operatorname{Int} \operatorname{cell}(G, i, 0) \cup \{G_{i+1,1} - [0,1]\}$.

(57) If $1 \leq i$ and $i < \operatorname{len} G$, then $\mathcal{L}(\frac{1}{2} \cdot (G_{i,\operatorname{width} G} + G_{i+1,\operatorname{width} G}) + [0,1], G_{i,\operatorname{width} G} + [0,1]) \subseteq \operatorname{Int} \operatorname{cell}(G, i, \operatorname{width} G) \cup \{G_{i,\operatorname{width} G} + [0,1]\}$.

(58) If $1 \leq i$ and $i < \operatorname{len} G$, then $\mathcal{L}(\frac{1}{2} \cdot (G_{i,\operatorname{width} G} + G_{i+1,\operatorname{width} G}) + [0,1], G_{i+1,\operatorname{width} G} + [0,1]) \subseteq \operatorname{Int} \operatorname{cell}(G, i, \operatorname{width} G) \cup \{G_{i+1,\operatorname{width} G} + [0,1]\}$.

(59) $\mathcal{L}(G_{1,1} - [1,1], G_{1,1} - [1,0]) \subseteq \operatorname{Int} \operatorname{cell}(G, 0, 0) \cup \{G_{1,1} - [1,0]\}$.

(60) $\mathcal{L}(G_{\operatorname{len} G,1}+[1,-1], G_{\operatorname{len} G,1}+[1,0]) \subseteq \operatorname{Int}\operatorname{cell}(G, \operatorname{len} G, 0) \cup \{G_{\operatorname{len} G,1}+[1,0]\}$.

(61) $\mathcal{L}(G_{1,\operatorname{width} G}+[-1,1], G_{1,\operatorname{width} G}-[1,0]) \subseteq \operatorname{Int}\operatorname{cell}(G, 0, \operatorname{width} G) \cup \{G_{1,\operatorname{width} G}-[1,0]\}$.

(62) $\mathcal{L}(G_{\operatorname{len} G,\operatorname{width} G}+[1,1], G_{\operatorname{len} G,\operatorname{width} G}+[1,0]) \subseteq \operatorname{Int}\operatorname{cell}(G, \operatorname{len} G, \operatorname{width} G) \cup \{G_{\operatorname{len} G,\operatorname{width} G}+[1,0]\}$.

(63) $\mathcal{L}(G_{1,1}-[1,1], G_{1,1}-[0,1]) \subseteq \operatorname{Int}\operatorname{cell}(G, 0, 0) \cup \{G_{1,1}-[0,1]\}$.

(64) $\mathcal{L}(G_{\operatorname{len} G,1}+[1,-1], G_{\operatorname{len} G,1}-[0,1]) \subseteq \operatorname{Int}\operatorname{cell}(G, \operatorname{len} G, 0) \cup \{G_{\operatorname{len} G,1}-[0,1]\}$.

(65) $\mathcal{L}(G_{1,\operatorname{width} G}+[-1,1], G_{1,\operatorname{width} G}+[0,1]) \subseteq \operatorname{Int}\operatorname{cell}(G, 0, \operatorname{width} G) \cup \{G_{1,\operatorname{width} G}+[0,1]\}$.

(66) $\mathcal{L}(G_{\operatorname{len} G,\operatorname{width} G}+[1,1], G_{\operatorname{len} G,\operatorname{width} G}+[0,1]) \subseteq \operatorname{Int}\operatorname{cell}(G, \operatorname{len} G, \operatorname{width} G) \cup \{G_{\operatorname{len} G,\operatorname{width} G}+[0,1]\}$.

(67) Suppose $1 \leq i$ and $i < \operatorname{len} G$ and $1 \leq j$ and $j+1 < \operatorname{width} G$. Then $\mathcal{L}(\frac{1}{2} \cdot (G_{i,j}+G_{i+1,j+1}), \frac{1}{2} \cdot (G_{i,j+1}+G_{i+1,j+2})) \subseteq \operatorname{Int}\operatorname{cell}(G, i, j) \cup \operatorname{Int}\operatorname{cell}(G, i, j+1) \cup \{\frac{1}{2} \cdot (G_{i,j+1}+G_{i+1,j+1})\}$.

(68) Suppose $1 \leq j$ and $j < \operatorname{width} G$ and $1 \leq i$ and $i+1 < \operatorname{len} G$. Then $\mathcal{L}(\frac{1}{2} \cdot (G_{i,j}+G_{i+1,j+1}), \frac{1}{2} \cdot (G_{i+1,j}+G_{i+2,j+1})) \subseteq \operatorname{Int}\operatorname{cell}(G, i, j) \cup \operatorname{Int}\operatorname{cell}(G, i+1, j) \cup \{\frac{1}{2} \cdot (G_{i+1,j}+G_{i+1,j+1})\}$.

(69) If $1 \leq i$ and $i < \operatorname{len} G$ and $1 < \operatorname{width} G$, then $\mathcal{L}(\frac{1}{2} \cdot (G_{i,1}+G_{i+1,1})-[0,1], \frac{1}{2} \cdot (G_{i,1}+G_{i+1,2})) \subseteq \operatorname{Int}\operatorname{cell}(G, i, 0) \cup \operatorname{Int}\operatorname{cell}(G, i, 1) \cup \{\frac{1}{2} \cdot (G_{i,1}+G_{i+1,1})\}$.

(70) Suppose $1 \leq i$ and $i < \operatorname{len} G$ and $1 < \operatorname{width} G$. Then $\mathcal{L}(\frac{1}{2} \cdot (G_{i,\operatorname{width} G}+G_{i+1,\operatorname{width} G})+[0,1], \frac{1}{2} \cdot (G_{i,\operatorname{width} G}+G_{i+1,\operatorname{width} G-'1})) \subseteq \operatorname{Int}\operatorname{cell}(G, i, \operatorname{width} G -' 1) \cup \operatorname{Int}\operatorname{cell}(G, i, \operatorname{width} G) \cup \{\frac{1}{2} \cdot (G_{i,\operatorname{width} G}+G_{i+1,\operatorname{width} G})\}$.

(71) If $1 \leq j$ and $j < \operatorname{width} G$ and $1 < \operatorname{len} G$, then $\mathcal{L}(\frac{1}{2} \cdot (G_{1,j}+G_{1,j+1})-[1,0], \frac{1}{2} \cdot (G_{1,j}+G_{2,j+1})) \subseteq \operatorname{Int}\operatorname{cell}(G, 0, j) \cup \operatorname{Int}\operatorname{cell}(G, 1, j) \cup \{\frac{1}{2} \cdot (G_{1,j}+G_{1,j+1})\}$.

(72) Suppose $1 \leq j$ and $j < \operatorname{width} G$ and $1 < \operatorname{len} G$. Then $\mathcal{L}(\frac{1}{2} \cdot (G_{\operatorname{len} G,j}+G_{\operatorname{len} G,j+1})+[1,0], \frac{1}{2} \cdot (G_{\operatorname{len} G,j}+G_{\operatorname{len} G-'1,j+1})) \subseteq \operatorname{Int}\operatorname{cell}(G, \operatorname{len} G -' 1, j) \cup \operatorname{Int}\operatorname{cell}(G, \operatorname{len} G, j) \cup \{\frac{1}{2} \cdot (G_{\operatorname{len} G,j}+G_{\operatorname{len} G,j+1})\}$.

(73) If $1 < \operatorname{len} G$ and $1 \leq j$ and $j+1 < \operatorname{width} G$, then $\mathcal{L}(\frac{1}{2} \cdot (G_{1,j}+G_{1,j+1})-[1,0], \frac{1}{2} \cdot (G_{1,j+1}+G_{1,j+2})-[1,0]) \subseteq \operatorname{Int}\operatorname{cell}(G, 0, j) \cup \operatorname{Int}\operatorname{cell}(G, 0, j+1) \cup \{G_{1,j+1}-[1,0]\}$.

(74) Suppose $1 < \operatorname{len} G$ and $1 \leq j$ and $j+1 < \operatorname{width} G$. Then $\mathcal{L}(\frac{1}{2} \cdot (G_{\operatorname{len} G,j}+G_{\operatorname{len} G,j+1})+[1,0], \frac{1}{2} \cdot (G_{\operatorname{len} G,j+1}+G_{\operatorname{len} G,j+2})+[1,0]) \subseteq \operatorname{Int}\operatorname{cell}(G, \operatorname{len} G, j) \cup \operatorname{Int}\operatorname{cell}(G, \operatorname{len} G, j+1) \cup \{G_{\operatorname{len} G,j+1}+[1,0]\}$.

(75) If $1 < \operatorname{width} G$ and $1 \leq i$ and $i+1 < \operatorname{len} G$, then $\mathcal{L}(\frac{1}{2} \cdot (G_{i,1}+G_{i+1,1})-[0,1], \frac{1}{2} \cdot (G_{i+1,1}+G_{i+2,1})-[0,1]) \subseteq \operatorname{Int}\operatorname{cell}(G, i, 0) \cup \operatorname{Int}\operatorname{cell}(G, i+1, 0) \cup \{G_{i+1,1}-[0,1]\}$.

(76) Suppose $1 < \operatorname{width} G$ and $1 \leq i$ and $i+1 < \operatorname{len} G$. Then $\mathcal{L}(\frac{1}{2} \cdot (G_{i,\operatorname{width} G} + G_{i+1,\operatorname{width} G}) + [0,1], \frac{1}{2} \cdot (G_{i+1,\operatorname{width} G} + G_{i+2,\operatorname{width} G}) + [0,1]) \subseteq \operatorname{Int} \operatorname{cell}(G, i, \operatorname{width} G) \cup \operatorname{Int} \operatorname{cell}(G, i+1, \operatorname{width} G) \cup \{G_{i+1,\operatorname{width} G} + [0,1]\}$.

(77) If $1 < \operatorname{len} G$ and $1 < \operatorname{width} G$, then $\mathcal{L}(G_{1,1} - [1,1], \frac{1}{2} \cdot (G_{1,1} + G_{1,2}) - [1,0]) \subseteq \operatorname{Int} \operatorname{cell}(G, 0, 0) \cup \operatorname{Int} \operatorname{cell}(G, 0, 1) \cup \{G_{1,1} - [1,0]\}$.

(78) If $1 < \operatorname{len} G$ and $1 < \operatorname{width} G$, then $\mathcal{L}(G_{\operatorname{len} G,1} + [1,-1], \frac{1}{2} \cdot (G_{\operatorname{len} G,1} + G_{\operatorname{len} G,2}) + [1,0]) \subseteq \operatorname{Int} \operatorname{cell}(G, \operatorname{len} G, 0) \cup \operatorname{Int} \operatorname{cell}(G, \operatorname{len} G, 1) \cup \{G_{\operatorname{len} G,1} + [1,0]\}$.

(79) If $1 < \operatorname{len} G$ and $1 < \operatorname{width} G$, then $\mathcal{L}(G_{1,\operatorname{width} G} + [-1,1], \frac{1}{2} \cdot (G_{1,\operatorname{width} G} + G_{1,\operatorname{width} G -' 1}) - [1,0]) \subseteq \operatorname{Int} \operatorname{cell}(G, 0, \operatorname{width} G) \cup \operatorname{Int} \operatorname{cell}(G, 0, \operatorname{width} G -' 1) \cup \{G_{1,\operatorname{width} G} - [1,0]\}$.

(80) If $1 < \operatorname{len} G$ and $1 < \operatorname{width} G$, then $\mathcal{L}(G_{\operatorname{len} G,\operatorname{width} G} + [1,1], \frac{1}{2} \cdot (G_{\operatorname{len} G,\operatorname{width} G} + G_{\operatorname{len} G,\operatorname{width} G -' 1}) + [1,0]) \subseteq \operatorname{Int} \operatorname{cell}(G, \operatorname{len} G, \operatorname{width} G) \cup \operatorname{Int} \operatorname{cell}(G, \operatorname{len} G, \operatorname{width} G -' 1) \cup \{G_{\operatorname{len} G,\operatorname{width} G} + [1,0]\}$.

(81) If $1 < \operatorname{width} G$ and $1 < \operatorname{len} G$, then $\mathcal{L}(G_{1,1} - [1,1], \frac{1}{2} \cdot (G_{1,1} + G_{2,1}) - [0,1]) \subseteq \operatorname{Int} \operatorname{cell}(G, 0, 0) \cup \operatorname{Int} \operatorname{cell}(G, 1, 0) \cup \{G_{1,1} - [0,1]\}$.

(82) If $1 < \operatorname{width} G$ and $1 < \operatorname{len} G$, then $\mathcal{L}(G_{1,\operatorname{width} G} + [-1,1], \frac{1}{2} \cdot (G_{1,\operatorname{width} G} + G_{2,\operatorname{width} G}) + [0,1]) \subseteq \operatorname{Int} \operatorname{cell}(G, 0, \operatorname{width} G) \cup \operatorname{Int} \operatorname{cell}(G, 1, \operatorname{width} G) \cup \{G_{1,\operatorname{width} G} + [0,1]\}$.

(83) If $1 < \operatorname{width} G$ and $1 < \operatorname{len} G$, then $\mathcal{L}(G_{\operatorname{len} G,1} + [1,-1], \frac{1}{2} \cdot (G_{\operatorname{len} G,1} + G_{\operatorname{len} G -' 1,1}) - [0,1]) \subseteq \operatorname{Int} \operatorname{cell}(G, \operatorname{len} G, 0) \cup \operatorname{Int} \operatorname{cell}(G, \operatorname{len} G -' 1, 0) \cup \{G_{\operatorname{len} G,1} - [0,1]\}$.

(84) If $1 < \operatorname{width} G$ and $1 < \operatorname{len} G$, then $\mathcal{L}(G_{\operatorname{len} G,\operatorname{width} G} + [1,1], \frac{1}{2} \cdot (G_{\operatorname{len} G,\operatorname{width} G} + G_{\operatorname{len} G -' 1,\operatorname{width} G}) + [0,1]) \subseteq \operatorname{Int} \operatorname{cell}(G, \operatorname{len} G, \operatorname{width} G) \cup \operatorname{Int} \operatorname{cell}(G, \operatorname{len} G -' 1, \operatorname{width} G) \cup \{G_{\operatorname{len} G,\operatorname{width} G} + [0,1]\}$.

(85) If $1 \leq i$ and $i+1 \leq \operatorname{len} G$ and $1 \leq j$ and $j+1 \leq \operatorname{width} G$, then $\mathcal{L}(\frac{1}{2} \cdot (G_{i,j} + G_{i+1,j+1}), p)$ meets $\operatorname{Int} \operatorname{cell}(G, i, j)$.

(86) If $1 \leq i$ and $i+1 \leq \operatorname{len} G$, then $\mathcal{L}(p, \frac{1}{2} \cdot (G_{i,\operatorname{width} G} + G_{i+1,\operatorname{width} G}) + [0,1])$ meets $\operatorname{Int} \operatorname{cell}(G, i, \operatorname{width} G)$.

(87) If $1 \leq i$ and $i+1 \leq \operatorname{len} G$, then $\mathcal{L}(\frac{1}{2} \cdot (G_{i,1} + G_{i+1,1}) - [0,1], p)$ meets $\operatorname{Int} \operatorname{cell}(G, i, 0)$.

(88) If $1 \leq j$ and $j+1 \leq \operatorname{width} G$, then $\mathcal{L}(\frac{1}{2} \cdot (G_{1,j} + G_{1,j+1}) - [1,0], p)$ meets $\operatorname{Int} \operatorname{cell}(G, 0, j)$.

(89) If $1 \leq j$ and $j+1 \leq \operatorname{width} G$, then $\mathcal{L}(p, \frac{1}{2} \cdot (G_{\operatorname{len} G,j} + G_{\operatorname{len} G,j+1}) + [1,0])$ meets $\operatorname{Int} \operatorname{cell}(G, \operatorname{len} G, j)$.

(90) $\mathcal{L}(p, G_{1,1} - [1,1])$ meets $\operatorname{Int} \operatorname{cell}(G, 0, 0)$.

(91) $\mathcal{L}(p, G_{\operatorname{len} G,\operatorname{width} G} + [1,1])$ meets $\operatorname{Int} \operatorname{cell}(G, \operatorname{len} G, \operatorname{width} G)$.

(92) $\mathcal{L}(p, G_{1,\operatorname{width} G} + [-1,1])$ meets $\operatorname{Int} \operatorname{cell}(G, 0, \operatorname{width} G)$.

(93) $\mathcal{L}(p, G_{\operatorname{len} G,1} + [1,-1])$ meets $\operatorname{Int} \operatorname{cell}(G, \operatorname{len} G, 0)$.

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