FORMALIZED MATHEMATICS
Volume 5, Number 2, 1996
Warsaw University - Białystok

Categorial Categories and Slice Categories

Grzegorz Bancerek
Institute of Mathematics
Polish Academy of Sciences

Summary. By categorial categories we mean categories with categories as objects and morphisms of the form (C_1, C_2, F), where C_1 and C_2 are categories and F is a functor from C_1 into C_2.

MML Identifier: CAT_5.

The terminology and notation used here are introduced in the following articles: [14], [16], [9], [15], [11], [17], [2], [3], [5], [12], [10], [7], [6], [4], [8], [1], and [13].

1. Categories with Triple-like Morphisms

Let D_1, D_2, D be non empty sets and let x be an element of $[\![[\![D_1, D_2]\!], D]\!]$. Then $x_{\mathbf{1},\mathbf{1}}$ is an element of D_1. Then $x_{\mathbf{1},\mathbf{2}}$ is an element of D_2.

Let D_1, D_2 be non empty sets and let x be an element of $[\![D_1, D_2]\!]$. Then $x_{\mathbf{2}}$ is an element of D_2.

Next we state the proposition

(1) Let C, D be category structures. Suppose the category structure of $C =$ the category structure of D. If C is category-like, then D is category-like.

A category structure has triple-like morphisms if:

(Def.1) For every morphism f of it there exists a set x such that $f = \langle\langle \operatorname{dom} f, \operatorname{cod} f\rangle, x\rangle$.

One can verify that there exists a strict category has triple-like morphisms.

Next we state the proposition

(2) Let C be a category structure with triple-like morphisms and let f be a morphism of C. Then $\operatorname{dom} f = f_{\mathbf{1},1}$ and $\operatorname{cod} f = f_{\mathbf{1},\mathbf{2}}$ and $f = \langle\langle \operatorname{dom} f, \operatorname{cod} f\rangle, f_{\mathbf{2}}\rangle$.

ISSN 1426–2630

Let C be a category structure with triple-like morphisms and let f be a morphism of C. Then $f_{\mathbf{1},\mathbf{1}}$ is an object of C. Then $f_{\mathbf{1},\mathbf{2}}$ is an object of C.

In this article we present several logical schemes. The scheme *CatEx* concerns non empty sets $\mathcal{A}$, $\mathcal{B}$, a binary functor $\mathcal{F}$ yielding arbitrary, and a ternary predicate $\mathcal{P}$, and states that:

> There exists a strict category C with triple-like morphisms such that
> (i) the objects of $C = \mathcal{A}$,
> (ii) for all elements a, b of $\mathcal{A}$ and for every element f of $\mathcal{B}$ such that $\mathcal{P}[a, b, f]$ holds $\langle\langle a, b\rangle, f\rangle$ is a morphism of C,
> (iii) for every morphism m of C there exist elements a, b of $\mathcal{A}$ and there exists an element f of $\mathcal{B}$ such that $m = \langle\langle a, b\rangle, f\rangle$ and $\mathcal{P}[a, b, f]$, and
> (iv) for all morphisms m_1, m_2 of C and for all elements a_1, a_2, a_3 of $\mathcal{A}$ and for all elements f_1, f_2 of $\mathcal{B}$ such that $m_1 = \langle\langle a_1, a_2\rangle, f_1\rangle$ and $m_2 = \langle\langle a_2, a_3\rangle, f_2\rangle$ holds $m_2 \cdot m_1 = \langle\langle a_1, a_3\rangle, \mathcal{F}(f_2, f_1)\rangle$

provided the parameters meet the following requirements:

- For all elements a, b, c of $\mathcal{A}$ and for all elements f, g of $\mathcal{B}$ such that $\mathcal{P}[a, b, f]$ and $\mathcal{P}[b, c, g]$ holds $\mathcal{F}(g, f) \in \mathcal{B}$ and $\mathcal{P}[a, c, \mathcal{F}(g, f)]$,
- Let a be an element of $\mathcal{A}$. Then there exists an element f of $\mathcal{B}$ such that
  (i) $\mathcal{P}[a, a, f]$, and
  (ii) for every element b of $\mathcal{A}$ and for every element g of $\mathcal{B}$ holds if $\mathcal{P}[a, b, g]$, then $\mathcal{F}(g, f) = g$ and if $\mathcal{P}[b, a, g]$, then $\mathcal{F}(f, g) = g$,
- Let a, b, c, d be elements of $\mathcal{A}$ and let f, g, h be elements of $\mathcal{B}$. If $\mathcal{P}[a, b, f]$ and $\mathcal{P}[b, c, g]$ and $\mathcal{P}[c, d, h]$, then $\mathcal{F}(h, \mathcal{F}(g, f)) = \mathcal{F}(\mathcal{F}(h, g), f)$.

The scheme *CatUniq* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, a binary functor $\mathcal{F}$ yielding arbitrary, and a ternary predicate $\mathcal{P}$, and states that:

> Let C_1, C_2 be strict categories with triple-like morphisms. Suppose that
> (i) the objects of $C_1 = \mathcal{A}$,
> (ii) for all elements a, b of $\mathcal{A}$ and for every element f of $\mathcal{B}$ such that $\mathcal{P}[a, b, f]$ holds $\langle\langle a, b\rangle, f\rangle$ is a morphism of C_1,
> (iii) for every morphism m of C_1 there exist elements a, b of $\mathcal{A}$ and there exists an element f of $\mathcal{B}$ such that $m = \langle\langle a, b\rangle, f\rangle$ and $\mathcal{P}[a, b, f]$,
> (iv) for all morphisms m_1, m_2 of C_1 and for all elements a_1, a_2, a_3 of $\mathcal{A}$ and for all elements f_1, f_2 of $\mathcal{B}$ such that $m_1 = \langle\langle a_1, a_2\rangle, f_1\rangle$ and $m_2 = \langle\langle a_2, a_3\rangle, f_2\rangle$ holds $m_2 \cdot m_1 = \langle\langle a_1, a_3\rangle, \mathcal{F}(f_2, f_1)\rangle$,
> (v) the objects of $C_2 = \mathcal{A}$,
> (vi) for all elements a, b of $\mathcal{A}$ and for every element f of $\mathcal{B}$ such that $\mathcal{P}[a, b, f]$ holds $\langle\langle a, b\rangle, f\rangle$ is a morphism of C_2,
> (vii) for every morphism m of C_2 there exist elements a, b of $\mathcal{A}$ and there exists an element f of $\mathcal{B}$ such that $m = \langle\langle a, b\rangle, f\rangle$ and

$\mathcal{P}[a, b, f]$, and

(viii) for all morphisms m_1, m_2 of C_2 and for all elements a_1, a_2, a_3 of $\mathcal{A}$ and for all elements f_1, f_2 of $\mathcal{B}$ such that $m_1 = \langle\langle a_1, a_2\rangle, f_1\rangle$ and $m_2 = \langle\langle a_2, a_3\rangle, f_2\rangle$ holds $m_2 \cdot m_1 = \langle\langle a_1, a_3\rangle, \mathcal{F}(f_2, f_1)\rangle$.

Then $C_1 = C_2$

provided the parameters meet the following requirement:

- Let a be an element of $\mathcal{A}$. Then there exists an element f of $\mathcal{B}$ such that
  (i) $\mathcal{P}[a, a, f]$, and
  (ii) for every element b of $\mathcal{A}$ and for every element g of $\mathcal{B}$ holds if $\mathcal{P}[a, b, g]$, then $\mathcal{F}(g, f) = g$ and if $\mathcal{P}[b, a, g]$, then $\mathcal{F}(f, g) = g$.

The scheme *FunctorEx* concerns categories $\mathcal{A}$, $\mathcal{B}$, a unary functor $\mathcal{F}$ yielding an object of $\mathcal{B}$, and a unary functor $\mathcal{G}$ yielding a set, and states that:

There exists a functor F from $\mathcal{A}$ to $\mathcal{B}$ such that for every morphism f of $\mathcal{A}$ holds $F(f) = \mathcal{G}(f)$

provided the following conditions are met:

- Let f be a morphism of $\mathcal{A}$. Then $\mathcal{G}(f)$ is a morphism of $\mathcal{B}$ and for every morphism g of $\mathcal{B}$ such that $g = \mathcal{G}(f)$ holds $\mathrm{dom}\, g = \mathcal{F}(\mathrm{dom}\, f)$ and $\mathrm{cod}\, g = \mathcal{F}(\mathrm{cod}\, f)$,
- For every object a of $\mathcal{A}$ holds $\mathcal{G}(\mathrm{id}_a) = \mathrm{id}_{\mathcal{F}(a)}$,
- For all morphisms f_1, f_2 of $\mathcal{A}$ and for all morphisms g_1, g_2 of $\mathcal{B}$ such that $g_1 = \mathcal{G}(f_1)$ and $g_2 = \mathcal{G}(f_2)$ and $\mathrm{dom}\, f_2 = \mathrm{cod}\, f_1$ holds $\mathcal{G}(f_2 \cdot f_1) = g_2 \cdot g_1$.

We now state two propositions:

(3) Let C_1 be a category and let C_2 be a subcategory of C_1. Suppose C_1 is a subcategory of C_2. Then the category structure of C_1 = the category structure of C_2.

(4) For every category C and for every subcategory D of C holds every subcategory of D is a subcategory of C.

Let C_1, C_2 be categories. Let us assume that there exists a category C such that C_1 is a subcategory of C and C_2 is a subcategory of C. And let us assume that there exists an object o_1 of C_1 such that o_1 is an object of C_2. The functor $C_1 \cap C_2$ yields a strict category and is defined by the conditions (Def.2).

(Def.2) (i) The objects of $C_1 \cap C_2$ = (the objects of C_1) $\cap$ (the objects of C_2),

(ii) the morphisms of $C_1 \cap C_2$ = (the morphisms of C_1) $\cap$ (the morphisms of C_2),

(iii) the dom-map of $C_1 \cap C_2$ = (the dom-map of C_1) $\restriction$ (the morphisms of C_2),

(iv) the cod-map of $C_1 \cap C_2$ = (the cod-map of C_1) $\restriction$ (the morphisms of C_2),

(v) the composition of $C_1 \cap C_2$ = (the composition of C_1) $\restriction$ ([: the morphisms of C_2, the morphisms of C_2 :] **qua** set), and

(vi) the id-map of $C_1 \cap C_2$ = (the id-map of C_1) $\restriction$ (the objects of C_2).

In the sequel C is a category and C_1, C_2 are subcategories of C.

The following propositions are true:

(5) If (the objects of C_1) $\cap$ (the objects of C_2) $\neq \emptyset$, then $C_1 \cap C_2 = C_2 \cap C_1$.

(6) If (the objects of C_1) $\cap$ (the objects of C_2) $\neq \emptyset$, then $C_1 \cap C_2$ is a subcategory of C_1 and $C_1 \cap C_2$ is a subcategory of C_2.

Let C, D be categories and let F be a functor from C to D. The functor $\mathrm{Im}\, F$ yields a strict subcategory of D and is defined by the conditions (Def.3).

(Def.3) (i) The objects of $\mathrm{Im}\, F = \mathrm{rng}\, \mathrm{Obj}\, F$,

(ii) $\mathrm{rng}\, F \subseteq$ the morphisms of $\mathrm{Im}\, F$, and

(iii) for every subcategory E of D such that the objects of $E = \mathrm{rng}\, \mathrm{Obj}\, F$ and $\mathrm{rng}\, F \subseteq$ the morphisms of E holds $\mathrm{Im}\, F$ is a subcategory of E.

Next we state three propositions:

(7) Let C, D be categories, and let E be a subcategory of D, and let F be a functor from C to D. If $\mathrm{rng}\, F \subseteq$ the morphisms of E, then F is a functor from C to E.

(8) For all categories C, D holds every functor from C to D is a functor from C to $\mathrm{Im}\, F$.

(9) Let C, D be categories, and let E be a subcategory of D, and let F be a functor from C to E, and let G be a functor from C to D. If $F = G$, then $\mathrm{Im}\, F = \mathrm{Im}\, G$.

2. Categorial Categories

A set is categorial if:

(Def.4) For every set x such that $x \in$ it holds x is a category.

One can check that there exists a non empty set which is categorial. Let us observe that a non empty set is categorial if:

(Def.5) Every element of it is a category.

A category is categorial if it satisfies the conditions (Def.6).

(Def.6) (i) The objects of it is categorial,

(ii) for every object a of it and for every category A such that $a = A$ holds $\mathrm{id}_a = \langle\langle A, A\rangle, \mathrm{id}_A\rangle$,

(iii) for every morphism m of it and for all categories A, B such that $A = \mathrm{dom}\, m$ and $B = \mathrm{cod}\, m$ there exists a functor F from A to B such that $m = \langle\langle A, B\rangle, F\rangle$, and

(iv) for all morphisms m_1, m_2 of it and for all categories A, B, C and for every functor F from A to B and for every functor G from B to C such that $m_1 = \langle\langle A, B\rangle, F\rangle$ and $m_2 = \langle\langle B, C\rangle, G\rangle$ holds $m_2 \cdot m_1 = \langle\langle A, C\rangle, G \cdot F\rangle$.

Let us mention that every category which is categorial has triple-like morphisms.

One can prove the following two propositions:

(10) Let C, D be categories. Suppose the category structure of C = the category structure of D. If C is categorial, then D is categorial.

(11) For every category C holds $\dot{\circlearrowleft}(C, \langle\langle C, C\rangle, \mathrm{id}_C\rangle)$ is categorial.

Let us note that there exists a strict category which is categorial.

We now state two propositions:

(12) For every categorial category C holds every object of C is a category.

(13) For every categorial category C and for every morphism f of C holds $\mathrm{dom}\, f = f_{\mathbf{1},\mathbf{1}}$ and $\mathrm{cod}\, f = f_{\mathbf{1},\mathbf{2}}$.

Let C be a categorial category and let m be a morphism of C. Then $m_{\mathbf{1},\mathbf{1}}$ is a category. Then $m_{\mathbf{1},\mathbf{2}}$ is a category.

We now state the proposition

(14) Let C_1, C_2 be categorial categories. Suppose the objects of C_1 = the objects of C_2 and the morphisms of C_1 = the morphisms of C_2. Then the category structure of C_1 = the category structure of C_2.

Let C be a categorial category. One can check that every subcategory of C is categorial.

We now state the proposition

(15) Let C, D be categorial categories. Suppose the morphisms of $C \subseteq$ the morphisms of D. Then C is a subcategory of D.

Let a be a set. Let us assume that a is a category. The functor $\mathrm{cat}\, a$ yields a category and is defined by:

(Def.7) $\mathrm{cat}\, a = a$.

One can prove the following proposition

(16) For every categorial category C and for every object c of C holds $\mathrm{cat}\, c = c$.

Let C be a categorial category and let m be a morphism of C. Then $m_{\mathbf{2}}$ is a functor from $\mathrm{cat}\,\mathrm{dom}\, m$ to $\mathrm{cat}\,\mathrm{cod}\, m$.

Next we state two propositions:

(17) Let X be a categorial non empty set and let Y be a non empty set. Suppose that
 (i) for all elements A, B, C of X and for every functor F from A to B and for every functor G from B to C such that $F \in Y$ and $G \in Y$ holds $G \cdot F \in Y$, and
 (ii) for every element A of X holds $\mathrm{id}_A \in Y$.

 Then there exists a strict categorial category C such that
 (iii) the objects of $C = X$, and
 (iv) for all elements A, B of X and for every functor F from A to B holds $\langle\langle A, B\rangle, F\rangle$ is a morphism of C iff $F \in Y$.

(18) Let X be a categorial non empty set, and let Y be a non empty set, and let C_1, C_2 be strict categorial categories. Suppose that
 (i) the objects of $C_1 = X$,

(ii) for all elements A, B of X and for every functor F from A to B holds $\langle\langle A, B\rangle, F\rangle$ is a morphism of C_1 iff $F \in Y$,
(iii) the objects of $C_2 = X$, and
(iv) for all elements A, B of X and for every functor F from A to B holds $\langle\langle A, B\rangle, F\rangle$ is a morphism of C_2 iff $F \in Y$.
Then $C_1 = C_2$.

A categorial category is full if it satisfies the condition (Def.8).

(Def.8) Let a, b be categories. Suppose a is an object of it and b is an object of it. Let F be a functor from a to b. Then $\langle\langle a, b\rangle, F\rangle$ is a morphism of it.

Let us note that there exists a categorial strict category which is full.

The following propositions are true:

(19) Let C_1, C_2 be full categorial categories. Suppose the objects of $C_1 =$ the objects of C_2. Then the category structure of $C_1 =$ the category structure of C_2.

(20) For every categorial non empty set A there exists a full categorial strict category C such that the objects of $C = A$.

(21) Let C be a categorial category and let D be a full categorial category. Suppose the objects of $C \subseteq$ the objects of D. Then C is a subcategory of D.

(22) Let C be a category, and let D_1, D_2 be categorial categories, and let F_1 be a functor from C to D_1, and let F_2 be a functor from C to D_2. If $F_1 = F_2$, then $\operatorname{Im} F_1 = \operatorname{Im} F_2$.

3. Slice Categories

Let C be a category and let o be an object of C. The functor $\operatorname{Hom}(o)$ yielding a non empty subset of the morphisms of C is defined by:

(Def.9) $\operatorname{Hom}(o) = (\text{the cod-map of } C)^{-1}\{o\}$.

The functor $\hom(o, \Box)$ yields a non empty subset of the morphisms of C and is defined by:

(Def.10) $\hom(o, \Box) = (\text{the dom-map of } C)^{-1}\{o\}$.

We now state several propositions:

(23) For every category C and for every object a of C and for every morphism f of C holds $f \in \operatorname{Hom}(a)$ iff $\operatorname{cod} f = a$.

(24) For every category C and for every object a of C and for every morphism f of C holds $f \in \hom(a, \Box)$ iff $\operatorname{dom} f = a$.

(25) For every category C and for all objects a, b of C holds $\hom(a, b) = \hom(a, \Box) \cap \operatorname{Hom}(b)$.

(26) For every category C and for every morphism f of C holds $f \in \hom(\operatorname{dom} f, \Box)$ and $f \in \operatorname{Hom}(\operatorname{cod} f)$.

(27) For every category C and for every morphism f of C and for every element g of $\mathrm{Hom}(\mathrm{dom}\, f)$ holds $f \cdot g \in \mathrm{Hom}(\mathrm{cod}\, f)$.

(28) For every category C and for every morphism f of C and for every element g of $\mathrm{hom}(\mathrm{cod}\, f, \Box)$ holds $g \cdot f \in \mathrm{hom}(\mathrm{dom}\, f, \Box)$.

Let C be a category and let o be an object of C. The functor $\mathrm{SliceCat}(C, o)$ yields a strict category with triple-like morphisms and is defined by the conditions (Def.11).

(Def.11) (i) The objects of $\mathrm{SliceCat}(C, o) = \mathrm{Hom}(o)$,

(ii) for all elements a, b of $\mathrm{Hom}(o)$ and for every morphism f of C such that $\mathrm{dom}\, b = \mathrm{cod}\, f$ and $a = b \cdot f$ holds $\langle\langle a, b\rangle, f\rangle$ is a morphism of $\mathrm{SliceCat}(C, o)$,

(iii) for every morphism m of $\mathrm{SliceCat}(C, o)$ there exist elements a, b of $\mathrm{Hom}(o)$ and there exists a morphism f of C such that $m = \langle\langle a, b\rangle, f\rangle$ and $\mathrm{dom}\, b = \mathrm{cod}\, f$ and $a = b \cdot f$, and

(iv) for all morphisms m_1, m_2 of $\mathrm{SliceCat}(C, o)$ and for all elements a_1, a_2, a_3 of $\mathrm{Hom}(o)$ and for all morphisms f_1, f_2 of C such that $m_1 = \langle\langle a_1, a_2\rangle, f_1\rangle$ and $m_2 = \langle\langle a_2, a_3\rangle, f_2\rangle$ holds $m_2 \cdot m_1 = \langle\langle a_1, a_3\rangle, f_2 \cdot f_1\rangle$.

The functor $\mathrm{SliceCat}(o, C)$ yielding a strict category with triple-like morphisms is defined by the conditions (Def.12).

(Def.12) (i) The objects of $\mathrm{SliceCat}(o, C) = \mathrm{hom}(o, \Box)$,

(ii) for all elements a, b of $\mathrm{hom}(o, \Box)$ and for every morphism f of C such that $\mathrm{dom}\, f = \mathrm{cod}\, a$ and $f \cdot a = b$ holds $\langle\langle a, b\rangle, f\rangle$ is a morphism of $\mathrm{SliceCat}(o, C)$,

(iii) for every morphism m of $\mathrm{SliceCat}(o, C)$ there exist elements a, b of $\mathrm{hom}(o, \Box)$ and there exists a morphism f of C such that $m = \langle\langle a, b\rangle, f\rangle$ and $\mathrm{dom}\, f = \mathrm{cod}\, a$ and $f \cdot a = b$, and

(iv) for all morphisms m_1, m_2 of $\mathrm{SliceCat}(o, C)$ and for all elements a_1, a_2, a_3 of $\mathrm{hom}(o, \Box)$ and for all morphisms f_1, f_2 of C such that $m_1 = \langle\langle a_1, a_2\rangle, f_1\rangle$ and $m_2 = \langle\langle a_2, a_3\rangle, f_2\rangle$ holds $m_2 \cdot m_1 = \langle\langle a_1, a_3\rangle, f_2 \cdot f_1\rangle$.

Let C be a category, let o be an object of C, and let m be a morphism of $\mathrm{SliceCat}(C, o)$. Then $m_{\mathbf{2}}$ is a morphism of C. Then $m_{\mathbf{1},\mathbf{1}}$ is an element of $\mathrm{Hom}(o)$. Then $m_{\mathbf{1},\mathbf{2}}$ is an element of $\mathrm{Hom}(o)$.

We now state two propositions:

(29) Let C be a category, and let a be an object of C, and let m be a morphism of $\mathrm{SliceCat}(C, a)$. Then $m = \langle\langle m_{\mathbf{1},\mathbf{1}}, m_{\mathbf{1},\mathbf{2}}\rangle, m_{\mathbf{2}}\rangle$ and $\mathrm{dom}(m_{\mathbf{1},\mathbf{2}}) = \mathrm{cod}(m_{\mathbf{2}})$ and $m_{\mathbf{1},\mathbf{1}} = m_{\mathbf{1},\mathbf{2}} \cdot m_{\mathbf{2}}$ and $\mathrm{dom}\, m = m_{\mathbf{1},\mathbf{1}}$ and $\mathrm{cod}\, m = m_{\mathbf{1},\mathbf{2}}$.

(30) Let C be a category, and let o be an object of C, and let f be an element of $\mathrm{Hom}(o)$, and let a be an object of $\mathrm{SliceCat}(C, o)$. If $a = f$, then $\mathrm{id}_a = \langle\langle a, a\rangle, \mathrm{id}_{\mathrm{dom}\, f}\rangle$.

Let C be a category, let o be an object of C, and let m be a morphism of $\mathrm{SliceCat}(o, C)$. Then $m_{\mathbf{2}}$ is a morphism of C. Then $m_{\mathbf{1},\mathbf{1}}$ is an element of $\mathrm{hom}(o, \Box)$. Then $m_{\mathbf{1},\mathbf{2}}$ is an element of $\mathrm{hom}(o, \Box)$.

We now state two propositions:

(31) Let C be a category, and let a be an object of C, and let m be a morphism of $\mathrm{SliceCat}(a, C)$. Then $m = \langle\langle m_{\mathbf{1},\mathbf{1}}, m_{\mathbf{1},\mathbf{2}}\rangle, m_{\mathbf{2}}\rangle$ and $\mathrm{dom}(m_{\mathbf{2}}) = \mathrm{cod}(m_{\mathbf{1},\mathbf{1}})$ and $m_{\mathbf{2}} \cdot m_{\mathbf{1},\mathbf{1}} = m_{\mathbf{1},\mathbf{2}}$ and $\mathrm{dom}\, m = m_{\mathbf{1},\mathbf{1}}$ and $\mathrm{cod}\, m = m_{\mathbf{1},\mathbf{2}}$.

(32) Let C be a category, and let o be an object of C, and let f be an element of $\mathrm{hom}(o, \Box)$, and let a be an object of $\mathrm{SliceCat}(o, C)$. If $a = f$, then $\mathrm{id}_a = \langle\langle a, a\rangle, \mathrm{id}_{\mathrm{cod}\, f}\rangle$.

4. Functors Between Slice Categories

Let C be a category and let f be a morphism of C. The functor $\mathrm{SliceFunctor}(f)$ yielding a functor from $\mathrm{SliceCat}(C, \mathrm{dom}\, f)$ to $\mathrm{SliceCat}(C, \mathrm{cod}\, f)$ is defined by:

(Def.13) For every morphism m of $\mathrm{SliceCat}(C, \mathrm{dom}\, f)$ holds $(\mathrm{SliceFunctor}(f))(m) = \langle\langle f \cdot m_{\mathbf{1},\mathbf{1}}, f \cdot m_{\mathbf{1},\mathbf{2}}\rangle, m_{\mathbf{2}}\rangle$.

The functor $\mathrm{SliceContraFunctor}(f)$ yields a functor from $\mathrm{SliceCat}(\mathrm{cod}\, f, C)$ to $\mathrm{SliceCat}(\mathrm{dom}\, f, C)$ and is defined as follows:

(Def.14) For every morphism m of $\mathrm{SliceCat}(\mathrm{cod}\, f, C)$ holds $(\mathrm{SliceContraFunctor}(f))(m) = \langle\langle m_{\mathbf{1},\mathbf{1}} \cdot f, m_{\mathbf{1},\mathbf{2}} \cdot f\rangle, m_{\mathbf{2}}\rangle$.

We now state two propositions:

(33) For every category C and for all morphisms f, g of C such that $\mathrm{dom}\, g = \mathrm{cod}\, f$ holds $\mathrm{SliceFunctor}(g \cdot f) = \mathrm{SliceFunctor}(g) \cdot \mathrm{SliceFunctor}(f)$.

(34) For every category C and for all morphisms f, g of C such that $\mathrm{dom}\, g = \mathrm{cod}\, f$ holds $\mathrm{SliceContraFunctor}(g \cdot f) = \mathrm{SliceContraFunctor}(f) \cdot \mathrm{SliceContraFunctor}(g)$.

References

[1] Grzegorz Bancerek and Agata Darmochwał. Comma category. *Formalized Mathematics*, 2(**5**):679–681, 1991.

[2] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.

[3] Czesław Byliński. Functions from a set to a set. *Formalized Mathematics*, 1(**1**):153–164, 1990.

[4] Czesław Byliński. Introduction to categories and functors. *Formalized Mathematics*, 1(**2**):409–420, 1990.

[5] Czesław Byliński. The modification of a function by a function and the iteration of the composition of a function. *Formalized Mathematics*, 1(**3**):521–527, 1990.

[6] Czesław Byliński. Opposite categories and contravariant functors. *Formalized Mathematics*, 2(**3**):419–424, 1991.

[7] Czesław Byliński. Partial functions. *Formalized Mathematics*, 1(**2**):357–367, 1990.

[8] Czesław Byliński. Subcategories and products of categories. *Formalized Mathematics*, 1(**4**):725–732, 1990.

[9] Beata Padlewska. Families of sets. *Formalized Mathematics*, 1(**1**):147–152, 1990.

[10] Andrzej Trybulec. Binary operations applied to functions. *Formalized Mathematics*, 1(**2**):329–334, 1990.

[11] Andrzej Trybulec. Domains and their Cartesian products. *Formalized Mathematics*, 1(**1**):115–122, 1990.

[12] Andrzej Trybulec. Function domains and Frænkel operator. *Formalized Mathematics*, 1(**3**):495–500, 1990.

[13] Andrzej Trybulec. Isomorphisms of categories. *Formalized Mathematics*, 2(**5**):629–634, 1991.

[14] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.

[15] Andrzej Trybulec. Tuples, projections and Cartesian products. *Formalized Mathematics*, 1(**1**):97–105, 1990.

[16] Zinaida Trybulec and Halina Święczkowska. Boolean properties of sets. *Formalized Mathematics*, 1(**1**):17–23, 1990.

[17] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1(**1**):73–83, 1990.

Received October 24, 1994