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Metric-Affine Configurations in Metric Affine Planes - Part I

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Summary. We introduce several configurational axioms for metric affine planes such as theorem on three perpendiculars, orthogonalization of major Desargues Axiom, orthogonalization of the trapezium variant of Desargues Axiom, axiom on parallel projection together with its indirect forms. For convenience we also consider affine Major Desargues Axiom. The aim is to prove logical relationships which hold between the introduced statements.

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The notation and terminology used here have been introduced in the following papers: [7], [8], [6], [3], [5], [4], [1], and [2]. We adopt the following rules: X will denote a metric affine plane and o, a, a_1, b, b_1, c, c_1 will denote elements of the points of X. Let us consider X. We say that Desargues Axiom holds in X if and only if the condition (Def.1) is satisfied.

(Def.1) Given o, a, a_1, b, b_1, c, c_1. Suppose that
- (i) $o \neq a$,
- (ii) $o \neq a_1$,
- (iii) $o \neq b$,
- (iv) $o \neq b_1$,
- (v) $o \neq c$,
- (vi) $o \neq c_1$,
- (vii) not $\mathbf{L}(b, b_1, a)$,
- (viii) not $\mathbf{L}(a, a_1, c)$,
- (ix) $\mathbf{L}(o, a, a_1)$,
- (x) $\mathbf{L}(o, b, b_1)$,
- (xi) $\mathbf{L}(o, c, c_1)$,
- (xii) $a, b \parallel a_1, b_1$,
- (xiii) $a, c \parallel a_1, c_1$.

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Then $b, c \parallel b_1, c_1$.

Let us consider X. We say that AH holds in X if and only if the condition (Def.2) is satisfied.

(Def.2) Given o, a, a_1, b, b_1, c, c_1. Suppose $o, a \perp o, a_1$ and $o, b \perp o, b_1$ and $o, c \perp o, c_1$ and $a, b \perp a_1, b_1$ and $o, a \parallel b, c$ and $a, c \perp a_1, c_1$ and $o, c \nparallel o, a$ and $o, a \nparallel o, b$. Then $b, c \perp b_1, c_1$.

Let us consider X. We say that theorem on three perpendiculars holds in X if and only if:

(Def.3) for all a, b, c such that not $\mathbf{L}(a, b, c)$ there exists an element d of the points of X such that $d, a \perp b, c$ and $d, b \perp a, c$ and $d, c \perp a, b$.

Let us consider X. We say that othogonal verion of Desargues Axiom holds in X if and only if the condition (Def.4) is satisfied.

(Def.4) Given o, a, a_1, b, b_1, c, c_1. Then if $o, a \perp o, a_1$ and $o, b \perp o, b_1$ and $o, c \perp o, c_1$ and $a, b \perp a_1, b_1$ and $a, c \perp a_1, c_1$ and $o, c \nparallel o, a$ and $o, a \nparallel o, b$, then $b, c \perp b_1, c_1$.

Let us consider X. We say that LIN holds in X if and only if the condition (Def.5) is satisfied.

(Def.5) Given o, a, a_1, b, b_1, c, c_1. Suppose that

(i) $o \neq a$,
(ii) $o \neq a_1$,
(iii) $o \neq b$,
(iv) $o \neq b_1$,
(v) $o \neq c$,
(vi) $o \neq c_1$,
(vii) $a \neq b$,
(viii) $o, c \perp o, c_1$,
(ix) $o, a \perp o, a_1$,
(x) $o, b \perp o, b_1$,
(xi) not $\mathbf{L}(o, c, a)$,
(xii) $\mathbf{L}(o, a, b)$,
(xiii) $\mathbf{L}(o, a_1, b_1)$,
(xiv) $a, c \perp a_1, c_1$,
(xv) $b, c \perp b_1, c_1$.

Then $a, a_1 \parallel b, b_1$.

Let us consider X. We say that first indirect form of LIN holds in X if and only if the condition (Def.6) is satisfied.

(Def.6) Given o, a, a_1, b, b_1, c, c_1. Suppose that

(i) $o \neq a$,
(ii) $o \neq a_1$,
(iii) $o \neq b$,
(iv) $o \neq b_1$,
(v) $o \neq c$,

(vi) $o \neq c_1$,
(vii) $a \neq b$,
(viii) $o, c \perp o, c_1$,
(ix) $o, a \perp o, a_1$,
(x) $o, b \perp o, b_1$,
(xi) not $\mathbf{L}(o, c, a)$,
(xii) $\mathbf{L}(o, a, b)$,
(xiii) $\mathbf{L}(o, a_1, b_1)$,
(xiv) $a, c \perp a_1, c_1$,
(xv) $a, a_1 \parallel b, b_1$.

Then $b, c \perp b_1, c_1$.

Let us consider X. We say that second indirect form of LIN holds in X if and only if the condition (Def.7) is satisfied.

(Def.7) Given o, a, a_1, b, b_1, c, c_1. Suppose that
(i) $o \neq a$,
(ii) $o \neq a_1$,
(iii) $o \neq b$,
(iv) $o \neq b_1$,
(v) $o \neq c$,
(vi) $o \neq c_1$,
(vii) $a \neq b$,
(viii) $a, a_1 \parallel b, b_1$,
(ix) $o, a \perp o, a_1$,
(x) $o, b \perp o, b_1$,
(xi) not $\mathbf{L}(o, c, a)$,
(xii) $\mathbf{L}(o, a, b)$,
(xiii) $\mathbf{L}(o, a_1, b_1)$,
(xiv) $a, c \perp a_1, c_1$,
(xv) $b, c \perp b_1, c_1$.

Then $o, c \perp o, c_1$.

We now state several propositions:

(1) If othogonal verion of Desargues Axiom holds in X, then Desargues Axiom holds in X.

(2) If othogonal verion of Desargues Axiom holds in X, then AH holds in X.

(3) If LIN holds in X, then first indirect form of LIN holds in X.

(4) If first indirect form of LIN holds in X, then second indirect form of LIN holds in X.

(5) If LIN holds in X, then othogonal verion of Desargues Axiom holds in X.

(6) If LIN holds in X, then theorem on three perpendiculars holds in X.

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