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Subgroup and Cosets of Subgroups

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Summary. We introduce notion of subgroup, coset of a subgroup, sets of left and right cosets of a subgroup. We define multiplication of two subset of a group, subset of reverse elemens of a group, intersection of two subgroups. We define the notion of an index of a subgroup and prove Lagrange theorem which states that in a finite group the order of the group equals the order of a subgroup multiplied by the index of the subgroup. Some theorems that belong rather to [1] are proved.

MML Identifier: GROUP_2.

The papers [9], [6], [3], [4], [1], [11], [10], [12], [5], [8], [7], and [2] provide the notation and terminology for this paper. Let D be a non-empty set. Then $\emptyset_D$ is a subset of D. Then Ω_D is a subset of D.

For simplicity we adopt the following convention: x is arbitrary, X, Y, Z are sets, k is a natural number, G, G_1, G_2, G_3 are groups, and a, b, g, g_1, g_2, h are elements of G. Let us consider G. A subset of G is a subset of the carrier of G.

In the sequel A, B, C denote subsets of G. The following propositions are true:

(1) If $x \in A$, then $x \in G$.

(2) If $x \in A$, then x is an element of G.

(3) If G is finite, then A is finite.

Let us consider G, A. The functor A^{-1} yielding a subset of G is defined by:

(Def.1) $A^{-1} = \{g^{-1} : g \in A\}$.

Next we state several propositions:

(4) $A^{-1} = \{g^{-1} : g \in A\}$.

(5) $x \in A^{-1}$ if and only if there exists g such that $x = g^{-1}$ and $g \in A$.

(6) $\{g\}^{-1} = \{g^{-1}\}$.

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(7) $\{g, h\}^{-1} = \{g^{-1}, h^{-1}\}$.

(8) $(\emptyset_{\text{the carrier of } G})^{-1} = \emptyset$.

(9) $(\Omega_{\text{the carrier of } G})^{-1} =$ the carrier of G.

(10) $A \neq \emptyset$ if and only if $A^{-1} \neq \emptyset$.

Let us consider G, A, B. The functor $A \cdot B$ yielding a subset of G is defined as follows:

(Def.2) $A \cdot B = \{g \cdot h : g \in A \wedge h \in B\}$.

One can prove the following propositions:

(11) $A \cdot B = \{g \cdot h : g \in A \wedge h \in B\}$.

(12) $x \in A \cdot B$ if and only if there exist g, h such that $x = g \cdot h$ and $g \in A$ and $h \in B$.

(13) $A \neq \emptyset$ and $B \neq \emptyset$ if and only if $A \cdot B \neq \emptyset$.

(14) $(A \cdot B) \cdot C = A \cdot (B \cdot C)$.

(15) $(A \cdot B)^{-1} = B^{-1} \cdot A^{-1}$.

(16) $A \cdot (B \cup C) = A \cdot B \cup A \cdot C$.

(17) $(A \cup B) \cdot C = A \cdot C \cup B \cdot C$.

(18) $A \cdot (B \cap C) \subseteq (A \cdot B) \cap (A \cdot C)$.

(19) $(A \cap B) \cdot C \subseteq (A \cdot C) \cap (B \cdot C)$.

(20) $\emptyset_{\text{the carrier of } G} \cdot A = \emptyset$ and $A \cdot \emptyset_{\text{the carrier of } G} = \emptyset$.

(21) If $A \neq \emptyset$, then $\Omega_{\text{the carrier of } G} \cdot A =$ the carrier of G and $A \cdot \Omega_{\text{the carrier of } G} =$ the carrier of G.

(22) $\{g\} \cdot \{h\} = \{g \cdot h\}$.

(23) $\{g\} \cdot \{g_1, g_2\} = \{g \cdot g_1, g \cdot g_2\}$.

(24) $\{g_1, g_2\} \cdot \{g\} = \{g_1 \cdot g, g_2 \cdot g\}$.

(25) $\{g, h\} \cdot \{g_1, g_2\} = \{g \cdot g_1, g \cdot g_2, h \cdot g_1, h \cdot g_2\}$.

(26) If for all g_1, g_2 such that $g_1 \in A$ and $g_2 \in A$ holds $g_1 \cdot g_2 \in A$ and for every g such that $g \in A$ holds $g^{-1} \in A$, then $A \cdot A = A$.

(27) If for every g such that $g \in A$ holds $g^{-1} \in A$, then $A^{-1} = A$.

(28) If for all a, b such that $a \in A$ and $b \in B$ holds $a \cdot b = b \cdot a$, then $A \cdot B = B \cdot A$.

(29) If G is an Abelian group, then $A \cdot B = B \cdot A$.

(30) If G is an Abelian group, then $(A \cdot B)^{-1} = A^{-1} \cdot B^{-1}$.

We now define two new functors. Let us consider G, g, A. The functor $g \cdot A$ yields a subset of G and is defined as follows:

(Def.3) $g \cdot A = \{g\} \cdot A$.

The functor $A \cdot g$ yielding a subset of G is defined as follows:

(Def.4) $A \cdot g = A \cdot \{g\}$.

Next we state a number of propositions:

(31) $g \cdot A = \{g\} \cdot A$.

(32) $A \cdot g = A \cdot \{g\}$.

(33) $x \in g \cdot A$ if and only if there exists h such that $x = g \cdot h$ and $h \in A$.

(34) $x \in A \cdot g$ if and only if there exists h such that $x = h \cdot g$ and $h \in A$.

(35) $(g \cdot A) \cdot B = g \cdot (A \cdot B)$.

(36) $(A \cdot g) \cdot B = A \cdot (g \cdot B)$.

(37) $(A \cdot B) \cdot g = A \cdot (B \cdot g)$.

(38) $(g \cdot h) \cdot A = g \cdot (h \cdot A)$.

(39) $(g \cdot A) \cdot h = g \cdot (A \cdot h)$.

(40) $(A \cdot g) \cdot h = A \cdot (g \cdot h)$.

(41) $\emptyset_{\text{the carrier of } G} \cdot a = \emptyset$ and $a \cdot \emptyset_{\text{the carrier of } G} = \emptyset$.

(42) $\Omega_{\text{the carrier of } G} \cdot a =$ the carrier of G and $a \cdot \Omega_{\text{the carrier of } G} =$ the carrier of G.

(43) $(1_G) \cdot A = A$ and $A \cdot (1_G) = A$.

(44) If G is an Abelian group, then $g \cdot A = A \cdot g$.

Let us consider G. A group is said to be a subgroup of G if:

(Def.5) the carrier of it $\subseteq$ the carrier of G and the operation of it $=$ (the operation of G) $\restriction$ [: the carrier of it, the carrier of it :].

One can prove the following proposition

(45) If the carrier of $G_1 \subseteq$ the carrier of G_2 and the operation of $G_1 =$ (the operation of G_2) $\restriction$ [: the carrier of G_1, the carrier of G_1 :], then G_1 is a subgroup of G_2.

We follow the rules: I, H, H_1, H_2, H_3 will be subgroups of G and h, h_1, h_2 will be elements of H. One can prove the following propositions:

(46) The carrier of $H \subseteq$ the carrier of G.

(47) The operation of $H =$ (the operation of G) $\restriction$ [: the carrier of H, the carrier of H :].

(48) If G is finite, then H is finite.

(49) If $x \in H$, then $x \in G$.

(50) $h \in G$.

(51) h is an element of G.

(52) If $h_1 = g_1$ and $h_2 = g_2$, then $h_1 \cdot h_2 = g_1 \cdot g_2$.

(53) $1_H = 1_G$.

(54) $1_{H_1} = 1_{H_2}$.

(55) $1_G \in H$.

(56) $1_{H_1} \in H_2$.

(57) If $h = g$, then $h^{-1} = g^{-1}$.

(58) $\cdot_H^{-1} = \cdot_G^{-1} \restriction$ (the carrier of H).

(59) If $g_1 \in H$ and $g_2 \in H$, then $g_1 \cdot g_2 \in H$.

(60) If $g \in H$, then $g^{-1} \in H$.

(61) If $A \neq \emptyset$ and for all g_1, g_2 such that $g_1 \in A$ and $g_2 \in A$ holds $g_1 \cdot g_2 \in A$ and for every g such that $g \in A$ holds $g^{-1} \in A$, then there exists H such that the carrier of $H = A$.

(62) If G is an Abelian group, then H is an Abelian group.

Let G be an Abelian group. We see that the subgroup of G is an Abelian group.

We now state several propositions:

(63) G is a subgroup of G.

(64) If G_1 is a subgroup of G_2 and G_2 is a subgroup of G_1, then $G_1 = G_2$.

(65) If G_1 is a subgroup of G_2 and G_2 is a subgroup of G_3, then G_1 is a subgroup of G_3.

(66) If the carrier of $H_1 \subseteq$ the carrier of H_2, then H_1 is a subgroup of H_2.

(67) If for every g such that $g \in H_1$ holds $g \in H_2$, then H_1 is a subgroup of H_2.

(68) If the carrier of $H_1 =$ the carrier of H_2, then $H_1 = H_2$.

(69) If for every g holds $g \in H_1$ if and only if $g \in H_2$, then $H_1 = H_2$.

Let us consider G, H_1, H_2. Let us note that one can characterize the predicate $H_1 = H_2$ by the following (equivalent) condition:

(Def.6) for every g holds $g \in H_1$ if and only if $g \in H_2$.

The following two propositions are true:

(70) If the carrier of $H =$ the carrier of G, then $H = G$.

(71) If for every g holds $g \in H$, then $H = G$.

Let us consider G. The functor $\{\mathbf{1}\}_G$ yields a subgroup of G and is defined by:

(Def.7) the carrier of $\{\mathbf{1}\}_G = \{1_G\}$.

Let us consider G. The functor Ω_G yielding a subgroup of G is defined as follows:

(Def.8) $\Omega_G = G$.

The following propositions are true:

(72) If the carrier of $H = \{1_G\}$, then $H = \{\mathbf{1}\}_G$.

(73) The carrier of $\{\mathbf{1}\}_G = \{1_G\}$.

(74) $\Omega_G = G$.

(75) $\{\mathbf{1}\}_H = \{\mathbf{1}\}_G$.

(76) $\{\mathbf{1}\}_{H_1} = \{\mathbf{1}\}_{H_2}$.

(77) $\{\mathbf{1}\}_G$ is a subgroup of H.

(78) H is a subgroup of Ω_G.

(79) G is a subgroup of Ω_G.

(80) $\{\mathbf{1}\}_G$ is finite.

(81) $\mathrm{ord}(\{\mathbf{1}\}_G) = 1$.
(82) If H is finite and $\mathrm{ord}(H) = 1$, then $H = \{\mathbf{1}\}_G$.
(83) $\mathrm{Ord}(H) \leq \mathrm{Ord}(G)$.
(84) If G is finite, then $\mathrm{ord}(H) \leq \mathrm{ord}(G)$.
(85) If G is finite and $\mathrm{ord}(G) = \mathrm{ord}(H)$, then $H = G$.

Let us consider G, H. The functor $\overline{H}$ yields a subset of G and is defined by:

(Def.9) $\overline{H} =$ the carrier of H.

The following propositions are true:

(86) $\overline{H} =$ the carrier of H.
(87) $\overline{H} \neq \emptyset$.
(88) $x \in \overline{H}$ if and only if $x \in H$.
(89) If $g_1 \in \overline{H}$ and $g_2 \in \overline{H}$, then $g_1 \cdot g_2 \in \overline{H}$.
(90) If $g \in \overline{H}$, then $g^{-1} \in \overline{H}$.
(91) $\overline{H} \cdot \overline{H} = \overline{H}$.
(92) $\overline{H}^{-1} = \overline{H}$.
(93) $\overline{H_1} \cdot \overline{H_2} = \overline{H_2} \cdot \overline{H_1}$ if and only if there exists H such that the carrier of $H = \overline{H_1} \cdot \overline{H_2}$.
(94) If G is an Abelian group, then there exists H such that the carrier of $H = \overline{H_1} \cdot \overline{H_2}$.

Let us consider G, H_1, H_2. The functor $H_1 \cap H_2$ yields a subgroup of G and is defined as follows:

(Def.10) the carrier of $H_1 \cap H_2 = \overline{H_1} \cap \overline{H_2}$.

One can prove the following propositions:

(95) If the carrier of $H = \overline{H_1} \cap \overline{H_2}$, then $H = H_1 \cap H_2$.
(96) The carrier of $H_1 \cap H_2 = \overline{H_1} \cap \overline{H_2}$.
(97) $H = H_1 \cap H_2$ if and only if the carrier of $H =$ (the carrier of H_1)$\cap$ (the carrier of H_2).
(98) $\overline{H_1 \cap H_2} = \overline{H_1} \cap \overline{H_2}$.
(99) $x \in H_1 \cap H_2$ if and only if $x \in H_1$ and $x \in H_2$.
(100) $H \cap H = H$.
(101) $H_1 \cap H_2 = H_2 \cap H_1$.
(102) $(H_1 \cap H_2) \cap H_3 = H_1 \cap (H_2 \cap H_3)$.
(103) $\{\mathbf{1}\}_G \cap H = \{\mathbf{1}\}_G$ and $H \cap \{\mathbf{1}\}_G = \{\mathbf{1}\}_G$.
(104) $H \cap \Omega_G = H$ and $\Omega_G \cap H = H$.
(105) $\Omega_G \cap \Omega_G = G$.
(106) $H_1 \cap H_2$ is a subgroup of H_1 and $H_1 \cap H_2$ is a subgroup of H_2.
(107) H_1 is a subgroup of H_2 if and only if $H_1 \cap H_2 = H_1$.
(108) If H_1 is a subgroup of H_2, then $H_1 \cap H_3$ is a subgroup of H_2.

(109) If H_1 is a subgroup of H_2 and H_1 is a subgroup of H_3, then H_1 is a subgroup of $H_2 \cap H_3$.

(110) If H_1 is a subgroup of H_2, then $H_1 \cap H_3$ is a subgroup of $H_2 \cap H_3$.

(111) If H_1 is finite or H_2 is finite, then $H_1 \cap H_2$ is finite.

We now define two new functors. Let us consider G, H, A. The functor $A \cdot H$ yielding a subset of G is defined as follows:

(Def.11) $A \cdot H = A \cdot \overline{H}$.

The functor $H \cdot A$ yields a subset of G and is defined as follows:

(Def.12) $H \cdot A = \overline{H} \cdot A$.

One can prove the following propositions:

(112) $A \cdot H = A \cdot \overline{H}$.

(113) $H \cdot A = \overline{H} \cdot A$.

(114) $x \in A \cdot H$ if and only if there exist g_1, g_2 such that $x = g_1 \cdot g_2$ and $g_1 \in A$ and $g_2 \in H$.

(115) $x \in H \cdot A$ if and only if there exist g_1, g_2 such that $x = g_1 \cdot g_2$ and $g_1 \in H$ and $g_2 \in A$.

(116) $(A \cdot B) \cdot H = A \cdot (B \cdot H)$.

(117) $(A \cdot H) \cdot B = A \cdot (H \cdot B)$.

(118) $(H \cdot A) \cdot B = H \cdot (A \cdot B)$.

(119) $(A \cdot H_1) \cdot H_2 = A \cdot (H_1 \cdot \overline{H_2})$.

(120) $(H_1 \cdot A) \cdot H_2 = H_1 \cdot (A \cdot H_2)$.

(121) $(H_1 \cdot \overline{H_2}) \cdot A = H_1 \cdot (H_2 \cdot A)$.

(122) If G is an Abelian group, then $A \cdot H = H \cdot A$.

We now define two new functors. Let us consider G, H, a. The functor $a \cdot H$ yielding a subset of G is defined as follows:

(Def.13) $a \cdot H = a \cdot \overline{H}$.

The functor $H \cdot a$ yielding a subset of G is defined by:

(Def.14) $H \cdot a = \overline{H} \cdot a$.

The following propositions are true:

(123) $a \cdot H = a \cdot \overline{H}$.

(124) $H \cdot a = \overline{H} \cdot a$.

(125) $x \in a \cdot H$ if and only if there exists g such that $x = a \cdot g$ and $g \in H$.

(126) $x \in H \cdot a$ if and only if there exists g such that $x = g \cdot a$ and $g \in H$.

(127) $(a \cdot b) \cdot H = a \cdot (b \cdot H)$.

(128) $(a \cdot H) \cdot b = a \cdot (H \cdot b)$.

(129) $(H \cdot a) \cdot b = H \cdot (a \cdot b)$.

(130) $a \in a \cdot H$ and $a \in H \cdot a$.

(131) $a \cdot H \neq \emptyset$ and $H \cdot a \neq \emptyset$.

(132) $(1_G) \cdot H = \overline{H}$ and $H \cdot (1_G) = \overline{H}$.

(133) $\{\mathbf{1}\}_G \cdot a = \{a\}$ and $a \cdot \{\mathbf{1}\}_G = \{a\}$.

(134) $a \cdot \Omega_G =$ the carrier of G and $\Omega_G \cdot a =$ the carrier of G.

(135) If G is an Abelian group, then $a \cdot H = H \cdot a$.

(136) $a \in H$ if and only if $a \cdot H = \overline{H}$.

(137) $a \cdot H = b \cdot H$ if and only if $b^{-1} \cdot a \in H$.

(138) $a \cdot H = b \cdot H$ if and only if $a \cdot H$ meets $b \cdot H$.

(139) $(a \cdot b) \cdot H \subseteq (a \cdot H) \cdot (b \cdot H)$.

(140) $\overline{H} \subseteq (a \cdot H) \cdot (a^{-1} \cdot H)$ and $\overline{H} \subseteq (a^{-1} \cdot H) \cdot (a \cdot H)$.

(141) $a^2 \cdot H \subseteq (a \cdot H) \cdot (a \cdot H)$.

(142) $a \in H$ if and only if $H \cdot a = \overline{H}$.

(143) $H \cdot a = H \cdot b$ if and only if $b \cdot a^{-1} \in H$.

(144) $H \cdot a = H \cdot b$ if and only if $H \cdot a$ meets $H \cdot b$.

(145) $(H \cdot a) \cdot b \subseteq (H \cdot a) \cdot (H \cdot b)$.

(146) $\overline{H} \subseteq (H \cdot a) \cdot (H \cdot a^{-1})$ and $\overline{H} \subseteq (H \cdot a^{-1}) \cdot (H \cdot a)$.

(147) $H \cdot a^2 \subseteq (H \cdot a) \cdot (H \cdot a)$.

(148) $a \cdot (H_1 \cap H_2) = (a \cdot H_1) \cap (a \cdot H_2)$.

(149) $(H_1 \cap H_2) \cdot a = (H_1 \cdot a) \cap (H_2 \cdot a)$.

(150) There exists H_1 such that the carrier of $H_1 = (a \cdot H_2) \cdot a^{-1}$.

(151) $a \cdot H \approx b \cdot H$.

(152) $a \cdot H \approx H \cdot b$.

(153) $H \cdot a \approx H \cdot b$.

(154) $\overline{H} \approx a \cdot H$ and $\overline{H} \approx H \cdot a$.

(155) $\mathrm{Ord}(H) = \overline{\overline{a \cdot H}}$ and $\mathrm{Ord}(H) = \overline{\overline{H \cdot a}}$.

(156) If H is finite, then $\mathrm{ord}(H) = \mathrm{card}(a \cdot H)$ and $\mathrm{ord}(H) = \mathrm{card}(H \cdot a)$.

The scheme *SubFamComp* deals with a set $\mathcal{A}$, a family $\mathcal{B}$ of subsets of $\mathcal{A}$, a family $\mathcal{C}$ of subsets of $\mathcal{A}$, and a unary predicate $\mathcal{P}$, and states that:

$\mathcal{B} = \mathcal{C}$

provided the parameters meet the following requirements:

- for every subset X of $\mathcal{A}$ holds $X \in \mathcal{B}$ if and only if $\mathcal{P}[X]$,
- for every subset X of $\mathcal{A}$ holds $X \in \mathcal{C}$ if and only if $\mathcal{P}[X]$.

We now define two new functors. Let us consider G, H. The left cosets of H yielding a family of subsets of the carrier of G is defined as follows:

(Def.15) $A \in$ the left cosets of H if and only if there exists a such that $A = a \cdot H$.

The right cosets of H yielding a family of subsets of the carrier of G is defined by:

(Def.16) $A \in$ the right cosets of H if and only if there exists a such that $A = H \cdot a$.

In the sequel F denotes a family of subsets of the carrier of G. One can prove the following propositions:

(157) If for every A holds $A \in F$ if and only if there exists a such that $A = a \cdot H$, then $F =$ the left cosets of H.

(158) If for every A holds $A \in F$ if and only if there exists a such that $A = H \cdot a$, then $F =$ the right cosets of H.

(159) $A \in$ the left cosets of H if and only if there exists a such that $A = a \cdot H$.

(160) $A \in$ the right cosets of H if and only if there exists a such that $A = H \cdot a$.

(161) If $x \in$ the left cosets of H or $x \in$ the right cosets of H, then x is a subset of G.

(162) $x \in$ the left cosets of H if and only if there exists a such that $x = a \cdot H$.

(163) $x \in$ the right cosets of H if and only if there exists a such that $x = H \cdot a$.

(164) If G is finite, then the right cosets of H is finite and the left cosets of H is finite.

(165) $\overline{H} \in$ the left cosets of H and $\overline{H} \in$ the right cosets of H.

(166) The left cosets of $H \approx$ the right cosets of H.

(167) $\bigcup$(The left cosets of H) = the carrier of G and $\bigcup$(the right cosets of H) = the carrier of G.

(168) The left cosets of $\{\mathbf{1}\}_G = \{\{a\}\}$.

(169) The right cosets of $\{\mathbf{1}\}_G = \{\{a\}\}$.

(170) If the left cosets of $H = \{\{a\}\}$, then $H = \{\mathbf{1}\}_G$.

(171) If the right cosets of $H = \{\{a\}\}$, then $H = \{\mathbf{1}\}_G$.

(172) The left cosets of $\Omega_G = \{$ the carrier of $G\}$ and the right cosets of $\Omega_G = \{$ the carrier of $G\}$.

(173) If the left cosets of $H = \{$ the carrier of $G\}$, then $H = G$.

(174) If the right cosets of $H = \{$ the carrier of $G\}$, then $H = G$.

Let us consider G, H. The functor $|\bullet : H|$ yielding a cardinal number is defined by:

(Def.17) $|\bullet : H| = \overline{\overline{\text{the left cosets of } H}}$.

We now state the proposition

(175) $|\bullet : H| = \overline{\overline{\text{the left cosets of } H}}$ and $|\bullet : H| = \overline{\overline{\text{the right cosets of } H}}$.

Let us consider G, H. Let us assume that the left cosets of H is finite. The functor $|\bullet : H|_{\mathbb{N}}$ yielding a natural number is defined as follows:

(Def.18) $|\bullet : H|_{\mathbb{N}} =$ card(the left cosets of H).

Next we state the proposition

(176) If the left cosets of H is finite, then $|\bullet : H|_{\mathbb{N}} =$ card(the left cosets of H) and $|\bullet : H|_{\mathbb{N}} =$ card(the right cosets of H).

Let D be a non-empty set, and let d be an element of D. Then $\{d\}$ is an element of Fin D.

The following two propositions are true:

(177) If G is finite, then $\mathrm{ord}(G) = \mathrm{ord}(H) \cdot |\bullet : H|_{\mathbb{N}}$.

(178) If G is finite, then $\mathrm{ord}(H) \mid \mathrm{ord}(G)$.

In the sequel J will denote a subgroup of H. One can prove the following propositions:

(179) If G is finite and $I = J$, then $|\bullet : I|_{\mathbb{N}} = |\bullet : J|_{\mathbb{N}} \cdot |\bullet : H|_{\mathbb{N}}$.

(180) $|\bullet : \Omega_G|_{\mathbb{N}} = 1$.

(181) If the left cosets of H is finite and $|\bullet : H|_{\mathbb{N}} = 1$, then $H = G$.

(182) $|\bullet : \{\mathbf{1}\}_G| = \mathrm{Ord}(G)$.

(183) If G is finite, then $|\bullet : \{\mathbf{1}\}_G|_{\mathbb{N}} = \mathrm{ord}(G)$.

(184) If G is finite and $|\bullet : H|_{\mathbb{N}} = \mathrm{ord}(G)$, then $H = \{\mathbf{1}\}_G$.

(185) If the left cosets of H is finite and $|\bullet : H| = \mathrm{Ord}(G)$, then G is finite and $H = \{\mathbf{1}\}_G$.

(186) If X is finite and for every Y such that $Y \in X$ holds Y is finite and $\mathrm{card}\, Y = k$ and for every Z such that $Z \in X$ and $Y \neq Z$ holds $Y \approx Z$ and Y misses Z, then $\mathrm{card}(\bigcup X) = k \cdot \mathrm{card}\, X$.

(187) If Y is finite and $X \subseteq Y$ and $\mathrm{card}\, X = \mathrm{card}\, Y$, then $X = Y$.

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