FORMALIZED MATHEMATICS
Vol.1, No.4, September–October 1990
Université Catholique de Louvain

# Translations in Affine Planes [1]

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**Summary.** Connections between Minor Desargues Axiom and the transitivity of translation groups are investigated. A formal proof of the theorem which establishes the equivalence of these two properties of affine planes is given. We also prove that, under additional requirement, the plane in question satisfies Fano Axiom; its translation group is uniquely two-divisible.

MML Identifier: `TRANSLAC`.

The terminology and notation used in this paper are introduced in the following papers: [1], [3], [4], [2], and [5]. We adopt the following rules: $AS$ is an affine space and $a$, $b$, $c$, $d$, $p$, $q$, $r$, $x$ are elements of the points of $AS$. Let us consider $AS$. We say that $AS$ satisfies Fano Axiom if and only if:

for all $a$, $b$, $c$, $d$ such that $a,b \parallel c,d$ and $a,c \parallel b,d$ and $a,d \parallel b,c$ holds $\mathbf{L}(a,b,c)$.

The following propositions are true:

(1) $AS$ satisfies Fano Axiom if and only if for all $a$, $b$, $c$, $d$ such that $a,b \parallel c,d$ and $a,c \parallel b,d$ and $a,d \parallel b,c$ holds $\mathbf{L}(a,b,c)$.

(2) If there exist $a$, $b$, $c$ such that $\mathbf{L}(a,b,c)$ and $a \neq b$ and $a \neq c$ and $b \neq c$, then for all $p$, $q$ such that $p \neq q$ there exists $r$ such that $\mathbf{L}(p,q,r)$ and $p \neq r$ and $q \neq r$.

(3) If there exist $a$, $b$ such that $a \neq b$ and for every $x$ such that $\mathbf{L}(a,b,x)$ holds $x = a$ or $x = b$, then for all $p$, $q$, $r$ such that $p \neq q$ and $\mathbf{L}(p,q,r)$ holds $r = p$ or $r = q$.

We follow a convention: $AFP$ is an affine plane, $a$, $a'$, $b$, $b'$, $c$, $c'$, $d$, $p$, $q$, $r$, $x$, $y$ are elements of the points of $AFP$, and $f$, $g$, $f_1$, $f_2$ are permutations of the points of $AFP$. We now state a number of propositions:

[1]Supported by RPBP.III-24.C2.




ISSN 0777–4028

(4) If $AFP$ satisfies Fano Axiom and $a, b \parallel c, d$ and $a, c \parallel b, d$ and not $\mathbf{L}(a, b, c)$,
then there exists $p$ such that $\mathbf{L}(b, c, p)$ and $\mathbf{L}(a, d, p)$.

(5) If $f$ is a translation and not $\mathbf{L}(a, f(a), x)$ and $a, f(a) \parallel x, y$ and $a, x \parallel f(a), y$, then $y = f(x)$.

(6) $AFP$ satisfies **des** if and only if for all $a$, $a'$, $b$, $c$, $b'$, $c'$ such that not $\mathbf{L}(a, a', b)$ and not $\mathbf{L}(a, a', c)$ and $a, a' \parallel b, b'$ and $a, a' \parallel c, c'$ and $a, b \parallel a', b'$ and $a, c \parallel a', c'$ holds $b, c \parallel b', c'$.

(7) There exists $f$ such that $f$ is a translation and $f(a) = a$.

(8) If for all $p$, $q$, $r$ such that $p \neq q$ and $\mathbf{L}(p, q, r)$ holds $r = p$ or $r = q$ and $a, b \parallel p, q$ and $a, p \parallel b, q$ and not $\mathbf{L}(a, b, p)$, then $a, q \parallel b, p$.

(9) If $AFP$ satisfies **des**, then there exists $f$ such that $f$ is a translation and $f(a) = b$.

(10) If for every $a$, $b$ there exists $f$ such that $f$ is a translation and $f(a) = b$, then $AFP$ satisfies **des**.

(11) If $f$ is a translation and $g$ is a translation and not $\mathbf{L}(a, f(a), g(a))$, then $f \cdot g = g \cdot f$.

(12) If $AFP$ satisfies **des** and $f$ is a translation and $g$ is a translation, then $f \cdot g = g \cdot f$.

(13) If $f$ is a translation and $g$ is a translation and $p, f(p) \parallel p, g(p)$, then $p, f(p) \parallel p, (f \cdot g)(p)$.

(14) If $AFP$ satisfies Fano Axiom and $AFP$ satisfies **des** and $f$ is a translation, then there exists $g$ such that $g$ is a translation and $g \cdot g = f$.

(15) If $AFP$ satisfies Fano Axiom and $f$ is a translation and
$f \cdot f = \mathrm{id}_{\text{the points of } AFP}$,
then $f = \mathrm{id}_{\text{the points of } AFP}$.

(16) If $AFP$ satisfies **des** and $AFP$ satisfies Fano Axiom and $g$ is a translation and $f_1$ is a translation and $f_2$ is a translation and $g = f_1 \cdot f_1$ and $g = f_2 \cdot f_2$, then $f_1 = f_2$.

## References

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*Received June 12, 1990*