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Some Properties of Functions Modul and Signum

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Summary. The article includes definitions and theorems concerning basic properties of the following functions : $|x|$ - modul of real number, sgn x - signum of real number.

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The article [1] provides the terminology and notation for this paper. In the sequel x, y, z, t are real numbers. Let us consider x. The functor $|x|$ yielding a real number, is defined by:

$|x| = x$ if $0 \leq x$, $|x| = -x$, otherwise.

One can prove the following propositions:

(1) If $0 \leq x$, then $|x| = x$.

(2) If $0 < x$, then $|x| = x$.

(3) If $0 \not\leq x$, then $|x| = -x$.

(4) If $x < 0$, then $|x| = -x$.

(5) $0 \leq |x|$.

(6) If $x \neq 0$, then $0 < |x|$.

(7) $x = 0$ if and only if $|x| = 0$.

(8) If $|x| = x$, then $0 \leq x$.

(9) If $|x| = -x$ and $x \neq 0$, then $x < 0$.

(10) For all x, y holds $|x \cdot y| = |x| \cdot |y|$.

(11) $-|x| \leq x$ and $x \leq |x|$.

(12) $-y \leq x$ and $x \leq y$ if and only if $|x| \leq y$.

(13) $|x + y| \leq |x| + |y|$.

(14) For every x such that $x \neq 0$ holds $|x| \cdot |\frac{1}{x}| = 1$.

(15) For every x such that $x \neq 0$ holds $|\frac{1}{x}| = \frac{1}{|x|}$.

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(16) For all x, y such that $y \neq 0$ holds $|\frac{x}{y}| = \frac{|x|}{|y|}$.

(17) $|x| = |-x|$.

(18) For all x, y holds $|x| - |y| \leq |x - y|$.

(19) For all x, y holds $|x - y| \leq |x| + |y|$.

(20) For every x holds $||x|| = |x|$.

(21) If $|x| \leq z$ and $|y| \leq t$, then $|x + y| \leq z + t$.

(22) $||x| - |y|| \leq |x - y|$.

(23) $y < |x|$ if and only if $x < -y$ or $y < x$.

(24) If $0 \leq x \cdot y$, then $|x + y| = |x| + |y|$.

(25) If $|x + y| = |x| + |y|$, then $0 \leq x \cdot y$.

(26) $\frac{|x+y|}{1+|x+y|} \leq \frac{|x|}{1+|x|} + \frac{|y|}{1+|y|}$.

Let us consider x. The functor $\operatorname{sgn} x$ yielding a real number, is defined by: $\operatorname{sgn} x = 1$ if $0 < x$, $\operatorname{sgn} x = -1$ if $x < 0$, $\operatorname{sgn} x = 0$, otherwise.

The following propositions are true:

(27) If $0 < x$, then $\operatorname{sgn} x = 1$.

(28) If $x < 0$, then $\operatorname{sgn} x = -1$.

(29) If $0 \not< x$ and $x \not< 0$, then $\operatorname{sgn} x = 0$.

(30) If $x = 0$, then $\operatorname{sgn} x = 0$.

(31) If $\operatorname{sgn} x = 1$, then $0 < x$.

(32) If $\operatorname{sgn} x = -1$, then $x < 0$.

(33) If $\operatorname{sgn} x = 0$, then $x = 0$.

(34) $x = |x| \cdot (\operatorname{sgn} x)$.

(35) $\operatorname{sgn}(x \cdot y) = (\operatorname{sgn} x) \cdot (\operatorname{sgn} y)$.

(36) $\operatorname{sgn}(\operatorname{sgn} x) = \operatorname{sgn} x$.

(37) $\operatorname{sgn}(x + y) \leq (\operatorname{sgn} x + \operatorname{sgn} y) + 1$.

(38) If $x \neq 0$, then $(\operatorname{sgn} x) \cdot (\operatorname{sgn} \frac{1}{x}) = 1$.

(39) If $x \neq 0$, then $\frac{1}{\operatorname{sgn} x} = \operatorname{sgn} \frac{1}{x}$.

(40) $(\operatorname{sgn} x + \operatorname{sgn} y) - 1 \leq \operatorname{sgn}(x + y)$.

(41) If $x \neq 0$, then $\operatorname{sgn} x = \operatorname{sgn} \frac{1}{x}$.

(42) If $y \neq 0$, then $\operatorname{sgn} \frac{x}{y} = \frac{\operatorname{sgn} x}{\operatorname{sgn} y}$.

References

[1] Krzysztof Hryniewiecki. Basic properties of real numbers. *Formalized Mathematics*, 1(**1**):35–40, 1990.

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