JOURNAL OF FORMALIZED MATHEMATICS
Volume 5, Released 1993, Published 2003
Inst. of Computer Science, Univ. of Białystok

Subspaces of Real Linear Space Generated by One, Two, or Three Vectors and Their Cosets

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MML Identifier: RLVECT_4.

WWW: http://mizar.org/JFM/Vol5/rlvect_4.html

The articles [5], [12], [7], [1], [2], [3], [4], [8], [9], [11], [10], and [6] provide the notation and terminology for this paper.

For simplicity, we adopt the following convention: x is a set, a, b, c are real numbers, V is a real linear space, u, v, v_1, v_2, v_3, w, w_1, w_2, w_3 are vectors of V, and W, W_1, W_2 are subspaces of V.

In this article we present several logical schemes. The scheme *LambdaSep3* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, elements $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$ of $\mathcal{A}$, elements $\mathcal{F}$, $\mathcal{G}$, $\mathcal{H}$ of $\mathcal{B}$, and a unary functor $\mathcal{F}$ yielding an element of $\mathcal{B}$, and states that:

> There exists a function f from $\mathcal{A}$ into $\mathcal{B}$ such that $f(\mathcal{C}) = \mathcal{F}$ and $f(\mathcal{D}) = \mathcal{G}$ and $f(\mathcal{E}) = \mathcal{H}$ and for every element C of $\mathcal{A}$ such that $C \neq \mathcal{C}$ and $C \neq \mathcal{D}$ and $C \neq \mathcal{E}$ holds $f(C) = \mathcal{F}(C)$

provided the following conditions are met:

- $\mathcal{C} \neq \mathcal{D}$,
- $\mathcal{C} \neq \mathcal{E}$, and
- $\mathcal{D} \neq \mathcal{E}$.

The scheme *LinCEx1* deals with a real linear space $\mathcal{A}$, a vector $\mathcal{B}$ of $\mathcal{A}$, and a real number $\mathcal{C}$, and states that:

> There exists a linear combination l of $\{\mathcal{B}\}$ such that $l(\mathcal{B}) = \mathcal{C}$

for all values of the parameters.

The scheme *LinCEx2* deals with a real linear space $\mathcal{A}$, vectors $\mathcal{B}$, $\mathcal{C}$ of $\mathcal{A}$, and real numbers $\mathcal{D}$, $\mathcal{E}$, and states that:

> There exists a linear combination l of $\{\mathcal{B}, \mathcal{C}\}$ such that $l(\mathcal{B}) = \mathcal{D}$ and $l(\mathcal{C}) = \mathcal{E}$

provided the parameters meet the following condition:

- $\mathcal{B} \neq \mathcal{C}$.

The scheme *LinCEx3* deals with a real linear space $\mathcal{A}$, vectors $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$ of $\mathcal{A}$, and real numbers $\mathcal{E}$, $\mathcal{F}$, $\mathcal{G}$, and states that:

> There exists a linear combination l of $\{\mathcal{B}, \mathcal{C}, \mathcal{D}\}$ such that $l(\mathcal{B}) = \mathcal{E}$ and $l(\mathcal{C}) = \mathcal{F}$ and $l(\mathcal{D}) = \mathcal{G}$

provided the following conditions are satisfied:

- $\mathcal{B} \neq \mathcal{C}$,
- $\mathcal{B} \neq \mathcal{D}$, and
- $\mathcal{C} \neq \mathcal{D}$.

One can prove the following propositions:

(1) $(v+w)-v=w$ and $(w+v)-v=w$ and $(v-v)+w=w$ and $(w-v)+v=w$ and $v+(w-v)=w$ and $w+(v-v)=w$ and $v-(v-w)=w$.

(2) $(v+u)-w=(v-w)+u$.

(4)[1] If $v_1-w=v_2-w$, then $v_1=v_2$.

(6)[2] $-a\cdot v=(-a)\cdot v$.

(7) If W_1 is a subspace of W_2, then $v+W_1 \subseteq v+W_2$.

(8) If $u \in v+W$, then $v+W=u+W$.

(9) For every linear combination l of $\{u,v,w\}$ such that $u \neq v$ and $u \neq w$ and $v \neq w$ holds $\sum l = l(u)\cdot u + l(v)\cdot v + l(w)\cdot w$.

(10) $u \neq v$ and $u \neq w$ and $v \neq w$ and $\{u,v,w\}$ is linearly independent if and only if for all a, b, c such that $a\cdot u+b\cdot v+c\cdot w=0_V$ holds $a=0$ and $b=0$ and $c=0$.

(11) $x \in \mathrm{Lin}(\{v\})$ iff there exists a such that $x=a\cdot v$.

(12) $v \in \mathrm{Lin}(\{v\})$.

(13) $x \in v+\mathrm{Lin}(\{w\})$ iff there exists a such that $x=v+a\cdot w$.

(14) $x \in \mathrm{Lin}(\{w_1,w_2\})$ iff there exist a, b such that $x=a\cdot w_1+b\cdot w_2$.

(15) $w_1 \in \mathrm{Lin}(\{w_1,w_2\})$ and $w_2 \in \mathrm{Lin}(\{w_1,w_2\})$.

(16) $x \in v+\mathrm{Lin}(\{w_1,w_2\})$ iff there exist a, b such that $x=v+a\cdot w_1+b\cdot w_2$.

(17) $x \in \mathrm{Lin}(\{v_1,v_2,v_3\})$ iff there exist a, b, c such that $x=a\cdot v_1+b\cdot v_2+c\cdot v_3$.

(18) $w_1 \in \mathrm{Lin}(\{w_1,w_2,w_3\})$ and $w_2 \in \mathrm{Lin}(\{w_1,w_2,w_3\})$ and $w_3 \in \mathrm{Lin}(\{w_1,w_2,w_3\})$.

(19) $x \in v+\mathrm{Lin}(\{w_1,w_2,w_3\})$ iff there exist a, b, c such that $x=v+a\cdot w_1+b\cdot w_2+c\cdot w_3$.

(20) If $\{u,v\}$ is linearly independent and $u \neq v$, then $\{u,v-u\}$ is linearly independent.

(21) If $\{u,v\}$ is linearly independent and $u \neq v$, then $\{u,v+u\}$ is linearly independent.

(22) If $\{u,v\}$ is linearly independent and $u \neq v$ and $a \neq 0$, then $\{u,a\cdot v\}$ is linearly independent.

(23) If $\{u,v\}$ is linearly independent and $u \neq v$, then $\{u,-v\}$ is linearly independent.

(24) If $a \neq b$, then $\{a\cdot v,b\cdot v\}$ is linearly dependent.

(25) If $a \neq 1$, then $\{v,a\cdot v\}$ is linearly dependent.

(26) If $\{u,w,v\}$ is linearly independent and $u \neq v$ and $u \neq w$ and $v \neq w$, then $\{u,w,v-u\}$ is linearly independent.

(27) If $\{u,w,v\}$ is linearly independent and $u \neq v$ and $u \neq w$ and $v \neq w$, then $\{u,w-u,v-u\}$ is linearly independent.

(28) If $\{u,w,v\}$ is linearly independent and $u \neq v$ and $u \neq w$ and $v \neq w$, then $\{u,w,v+u\}$ is linearly independent.

(29) If $\{u,w,v\}$ is linearly independent and $u \neq v$ and $u \neq w$ and $v \neq w$, then $\{u,w+u,v+u\}$ is linearly independent.

(30) If $\{u,w,v\}$ is linearly independent and $u \neq v$ and $u \neq w$ and $v \neq w$ and $a \neq 0$, then $\{u,w,a\cdot v\}$ is linearly independent.

[1] The proposition (3) has been removed.
[2] The proposition (5) has been removed.

(31) If $\{u, w, v\}$ is linearly independent and $u \neq v$ and $u \neq w$ and $v \neq w$ and $a \neq 0$ and $b \neq 0$, then $\{u, a \cdot w, b \cdot v\}$ is linearly independent.

(32) If $\{u, w, v\}$ is linearly independent and $u \neq v$ and $u \neq w$ and $v \neq w$, then $\{u, w, -v\}$ is linearly independent.

(33) If $\{u, w, v\}$ is linearly independent and $u \neq v$ and $u \neq w$ and $v \neq w$, then $\{u, -w, -v\}$ is linearly independent.

(34) If $a \neq b$, then $\{a \cdot v, b \cdot v, w\}$ is linearly dependent.

(35) If $a \neq 1$, then $\{v, a \cdot v, w\}$ is linearly dependent.

(36) If $v \in \mathrm{Lin}(\{w\})$ and $v \neq 0_V$, then $\mathrm{Lin}(\{v\}) = \mathrm{Lin}(\{w\})$.

(37) Suppose $v_1 \neq v_2$ and $\{v_1, v_2\}$ is linearly independent and $v_1 \in \mathrm{Lin}(\{w_1, w_2\})$ and $v_2 \in \mathrm{Lin}(\{w_1, w_2\})$. Then $\mathrm{Lin}(\{w_1, w_2\}) = \mathrm{Lin}(\{v_1, v_2\})$ and $\{w_1, w_2\}$ is linearly independent and $w_1 \neq w_2$.

(38) If $w \neq 0_V$ and $\{v, w\}$ is linearly dependent, then there exists a such that $v = a \cdot w$.

(39) If $v \neq w$ and $\{v, w\}$ is linearly independent and $\{u, v, w\}$ is linearly dependent, then there exist a, b such that $u = a \cdot v + b \cdot w$.

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Received February 24, 1993

Published January 2, 2004